Inelastic Capital*

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Abstract

How does the economy absorb a rise in desired saving? In the neoclassical growth model, long-run capital supply is perfectly elastic, so saving shocks are absorbed entirely by capital formation rather than by capital asset prices. We show that this result fails when capital formation is relatively labor-intensive or when workers have comparative advantage across sectors. We quantify our theory using sufficient statistics from the U.S. private sector (factor shares, sectoral labor composition) and find that higher capital prices absorb roughly 30% of long-run demand shocks. The reason is that investment disproportionately uses specialized workers, making their skills a bottleneck to capital accumulation.

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Introduction

The neoclassical growth model treats capital formation as a frictionless process: a single final good can be used either for consumption or to accumulate capital. A well-understood implication is that the supply curve for capital goods is perfectly elastic conditional on investment-specific technology. The relative price of capital goods may move over time with that technology, but capital demand cannot move it. As a result, demand shocks in the capital market (e.g., changes in household saving, taxes on capital, etc.) are entirely absorbed by real capital formation without raising the replacement cost of capital.

This prediction, however, appears to be at odds with the data, where capital asset prices are more volatile than quantities, even at long horizons. For instance, the literature finds that international capital flows, demographics, and rising inequality have contributed to the global saving glut of the 2000s (Bernanke, 2007; Auclert et al., 2021; Mian et al., 2024), which coincided with modest capital formation but large asset revaluations in the U.S. The q-theory of investment provides an apparent explanation for the disconnect between valuations and investment, but only in the short run, not in the medium and long run.

We take a different route and focus on how investment goods, including intangible capital, are produced. Our starting point is that producing capital is relatively labor-intensive and requires specialized labor whose aggregate supply is inherently limited.

Over 2020–2024, roughly 60% of aggregate U.S. investment consisted of intellectual property products and organizational services, both of which have high labor shares and require scientists, managers, and financiers. Even traditional investment sectors such as construction are much more labor-intensive than the economy as a whole and hire from a pool of experienced construction workers and managers. In both cases, scaling up aggregate investment would raise relative marginal costs because of a specialized-labor bottleneck.

Using a two-sector model in which factor shares differ across the consumption and investment sectors (Uzawa, 1961) and labor sorts on comparative advantage (Roy, 1951), we show that technology asymmetry and labor specialization make the aggregate capital supply curve upward-sloping: as the investment sector expands, its relative marginal cost rises. Capital-demand shocks are therefore absorbed partly by prices and partly by quantities.

We first formalize this idea by deriving long-run comparative-static formulas for how capital quantities and prices respond to an arbitrary demand shock (i.e., a shift in capital valuations that leaves technology unchanged). We then map those formulas to U.S. data and conclude that the world lies between the neoclassical growth model (perfectly elastic capital) and an endowment economy (inelastic capital). Quantitatively, the capital price response overshoots the demand shock on impact and absorbs roughly 30% of it in the long run. The medium-run capital response is roughly 50% smaller than in the neoclassical growth model.

Model. Section 1 calculates the long-run effects of a capital income tax shock on the capital stock and on the price of capital relative to consumption. We begin with a two-sector model (consumption and investment) with two factors (labor and capital). Production is Cobb–Douglas and labor supply is Roy–Fréchet. We show that, whenever (i) investment requires more labor than consumption or (ii) labor is imperfectly mobile across sectors, the cost of producing capital goods *rises* with aggregate investment (i.e., the capital supply curve is upward sloping). Expanding investment raises the price of the factor used most intensively by investment producers, thereby dampening capital accumulation.

Quantity and price responses are governed by two sets of sufficient statistics: (i) sectoral factor shares and (ii) the labor supply elasticity across sectors. We extend the theory to CES production, multiple capital types, dynamic occupation mobility, adjustment costs, imported inputs, and modified Euler equations. In each case we describe which additional information is needed to quantify the model (for instance, how factor shares respond to shocks when moving away from Cobb–Douglas).

Measurement. Section 2 combines the BEA's Make and Use Tables with production accounts and worker microdata to measure technology asymmetry and labor specialization in the United States. We divide private final demand into consumption and four investment goods: equipment, structures, intellectual property products, and brand/organizational services. Input–output accounting measures the labor embodied directly and indirectly in each good. Census data define each occupation bundle; CPS pairwise flows show how readily workers can move across them.

Two facts matter for our analysis. First, the labor share is higher in investment than in consumption, and the gap has widened as R&D and organizational services have expanded. Second, investment relies on occupation bundles with limited flow-based overlap with consumption. The quantitative model uses the same factor shares, occupation bundles, and CPS pairwise flows.

Quantitative predictions. Section 3 quantifies the propagation of demand shocks in an extension of the stylized model. We add three ingredients: (i) multiple types of labor and capital, (ii) occupation-specific labor mobility, and (iii) type-specific internal adjustment costs. We calibrate sectoral technology using our sectoral dataset and assumptions linking sectoral capital ownership to the use of capital services. We calibrate labor supply using CPS occupation-transition matrices and cross-elasticities together with external evidence on education choice.

The calibrated model has first-order implications for the incidence of macro shocks, cutting the quantity effect of a given demand shock by roughly half at a ten-year horizon (and about one quarter in the long run) relative to the neoclassical growth model. We decompose the long-run price effect into technology asymmetry, labor specialization, and capital-type composition terms. We then benchmark the model's split of nominal responses into prices and quantities against quasi-experimental evidence on investment tax incentives (Goolsbee, 1998b; House and Shapiro, 2008; House et al., 2017) and evidence on scientist-and-engineer wages and hours following federal R&D demand shifts (Goolsbee, 1998a).

Literature review. This paper revisits Uzawa (1961)'s insight that macroeconomic dynamics depend on technology asymmetry in the production of consumption versus investment. We consider a parsimonious departure from his model that can account for U.S. data—multiple types of capital and labor together with imperfect labor mobility—and calculate how capital-market demand shocks affect quantities and prices. Whether capital supply is elastic or inelastic matters for welfare incidence and optimal policy, a question going back to Harberger (1962). A large literature interprets movements in the relative price of investment goods as investment-specific technology shocks (Greenwood et al., 1997; Justiniano et al., 2011). In our model, demand shocks also move the relative price of investment goods along an upward-sloping supply curve.

Sectoral technology differences dampen the response of real investment to a demand shock, even in the long run. Section 2 maps this mechanism to U.S. data over 1963–2024. Investment is more labor-intensive than consumption, and the gap has grown with the rising importance of R&D and organizational capital. Earlier empirical work measuring labor-share differences between consumption and investment includes Chari et al. (1996), Cummins and Violante (2002), and Valentinyi and Herrendorf (2008). We extend the definition of investment goods to intangibles, including organizational capital (e.g., Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt, Falato and Xiaolan, 2023; Corrado, Haskel, Jona-Lasinio and Iommi, 2022).

We aggregate economic activity into broad expenditure sectors (consumption versus investment) rather than measuring the full production and investment network at the industry level (as in vom Lehn and Winberry, 2022; Casal and Caunedo, 2024; Baqaee and Malmberg, 2025). This choice reflects our focus on demand shocks, which raise demand for capital goods, rather than industry-level productivity shocks, which reallocate inputs across industries.

Finally, our theory is not about a disconnect between investment and q due to market power (see, e.g., Gutiérrez and Philippon, 2017; Farhi and Gourio, 2018; Crouzet and Eberly, 2023), but rather a standard technological force that makes the supply of capital goods inelastic at the aggregate level and dampens the real effect of demand shocks. Our analysis focuses on Walrasian forces (i.e., price adjustments and sectoral reallocation of finite resources). As discussed in Mussa (1977), our core channel, which works through equilibrium price adjustments, differs from models with capital adjustment costs (see, e.g., Lucas Jr, 1967; Uzawa, 1969; Hayashi, 1982).

1 Theory

We study a simple two-sector model with consumption and investment goods. We derive a sufficient-statistic formula for the long-run elasticities of capital prices and quantities to shocks in desired saving. Two forces determine long-run effects: the difference in the capital shares between sectors (which governs the ability to scale the technology), and the structure of comparative advantage in the labor market (which governs how easy it is to reallocate labor across sectors).

1.1 Stylized model

Production in each sector requires a combination of labor and capital. The model differs from the benchmark neoclassical growth model in two ways: the consumption and investment sectors have heterogeneous factor intensities and the sectoral labor supply is imperfectly elastic. We introduce these two ingredients because they capture the main forces omitted by the standard one-sector model that make the long-run supply of capital less than perfectly elastic.

Supply side. We consider an infinite-horizon economy populated by representative firms and households. We suppress time subscripts for clarity. There are two factors, capital K and labor L, and two final goods, C and I. Production in each sector is Cobb–Douglas with constant returns to scale:

$$C = A_C K_C^{\alpha_C} L_C^{1-\alpha_C}, \qquad \alpha_C \in (0,1), \tag{production–C sector}$$

$$I = A_I K_I^{\alpha_I} L_I^{1-\alpha_I}, \qquad \alpha_I \in (0,1). \tag{production–I sector}$$

Notice that the usual one-sector neoclassical growth model technology is nested when $\alpha_C = \alpha_I$. Empirically, we will show in the next section that the consumption sector has a higher capital intensity than the investment sector; that is, $\alpha_C > \alpha_I$. Investment goods are accumulated into the total stock of capital K, which depreciates at rate δ:

$$K' = (1-\delta)K + I, \qquad \delta \in (0,1). \tag{capital accumulation}$$

Demand side. Households have CRRA utility with elasticity of intertemporal substitution ψ and impatience parameter β. Consumption is the numeraire and we denote by p_K the price of the investment good relative to the consumption good. Households can hold capital or one-period bonds (in zero net supply) with interest rate r.

Household consumption choice implies the Euler equation:

$$\beta(1+r')\left(\frac{C'}{C}\right)^{-\frac{1}{\psi}} = 1. \tag{Euler}$$

We denote r_K the rental rate of capital and r the interest rate on one-period bonds. We introduce a wedge τ between the rental rate of capital and the cash flows that households receive (e.g., a capital income tax). The no-arbitrage condition between capital and bonds yields a valuation equation for the stock of capital:

$$(1+r')p_K = (1-\tau)r'_K + (1-\delta)p'_K. \tag{capital valuation}$$

Households supply specialized labor to both sectors. We denote w_C and w_I the wage in the consumption and investment sectors, and assume that the sectoral labor supply curves are upward-sloping due to comparative advantage. Formally, the representative household chooses a quantity of labor in each sector to maximize labor income $w_C L_C + w_I L_I$ subject to the constraint

$$1 = \left(\pi\, L_C^{\frac{1+\phi}{\phi}} + (1-\pi)\, L_I^{\frac{1+\phi}{\phi}}\right)^{\frac{\phi}{1+\phi}}, \qquad \pi \in (0,1),\ \phi > 0. \tag{labor supply}$$

The parameter ϕ corresponds to the elasticity of sectoral labor supply. The limit $\phi \downarrow 0$ gives fixed shares: labor cannot get reallocated across sectors. In contrast, the case $\phi \to \infty$ implies perfect mobility, i.e., $1 = \pi L_C + (1-\pi)L_I$.

This specification for sectoral labor supply can be micro-founded through a Roy–Fréchet selection model. Formally, consider a continuum of households that draw a productivity pair (a_C, a_I) according to a bivariate Fréchet distribution with shape $1+\phi$. Each household chooses the sector that maximizes labor income—$a_C w_C$ in consumption and $a_I w_I$ in investment. The resulting labor supply is equivalent to (labor supply) for a representative household (see, for instance, McFadden, 1974, Eaton and Kortum, 2002).

Equilibrium. An equilibrium consists of allocations and prices such that firms minimize costs, households choose consumption and sectoral labor supply, and markets clear. The capital-accumulation and capital-valuation equations also hold. Together with the Euler equation, these are the three intertemporal conditions. The remaining conditions constitute the intratemporal block, which pins down allocations and prices at each date given the state of the economy, summarized by p_K and K.

In the rest of the section, we focus on the long run to set intuition (see Appendix A.1 for a characterization of the transition dynamics). The steady-state equations are $C = C'$, $p_K = p'_K$, and $K = K'$. Plugging in the intertemporal equations yields the usual conditions:

$$r = 1/\beta - 1, \quad I = \delta K, \quad (r + \delta)p_K = (1 - \tau)r_K \qquad \text{(steady-state)}$$

The first equation says that the long-run interest rate is pinned down by the household's preference parameters. The second equation says that the long-run investment rate is pinned down by the depreciation rate of capital. The third states that the user cost of capital equals the after-tax return on capital.

Comparative statics. We are interested in how capital prices and quantities respond to a long-run shift in the demand for capital. The model experiment is a perturbation of the wedge $\mathrm{d}\log(1 - \tau)$ around a steady state with $\tau = 0$. The wedge τ affects equilibrium only through the capital valuation equation, so the experiment shifts capital demand without changing the supply side. It is isomorphic, up to a scale, to a change in the representative household's β (e.g., due to demographic change, wealth concentration, or financial repression) as well as a capital income tax.

In the neoclassical growth model, consumption and investment goods are produced with the same technology, so the supply side pins down the relative price of investment goods conditional on investment-specific technology. In steady state, (capital valuation) then implies that, for any decrease in the capital tax τ, the gross rental rate r_K decreases one-for-one so that the after-tax rental rate $r_K(1 - \tau)$ is unchanged. The capital stock then adjusts so that its marginal product equals this new rental rate. Hence, the long-run elasticity of the capital-good price to the demand shock is zero, and the long-run elasticity of the capital stock is $1/(1-\alpha)$, the inverse of one minus the Cobb–Douglas exponent on capital (i.e., the inverse of the labor share).

This frictionless limit is also the common-technology representation of investment-specific technical change in Greenwood et al. (1997). In their efficiency-unit notation, investment-specific productivity pins down the acquisition price of new capital relative to consumption. That relative price may therefore change over calendar time even though its response to a capital-demand shock is zero. Their two-sector representation is isomorphic to the one-sector model when the consumption- and investment-good sectors have equal factor shares. Our departure is precisely to allow those sectors to use different factor intensities and specialized workers.

In a multi-sector model, however, the technology side no longer pins down the relative price of capital. When desired saving increases, the economy reorients factors of production from the consumption to the investment sector, but that increases the cost of producing investment goods relative to consumption goods. As a result, the supply of capital is not perfectly elastic and some of the shock in the demand for capital is absorbed through higher capital prices rather than higher quantities.

The following proposition determines the long-run elasticities of capital prices and quantities in the stylized model. We denote by λ_C and λ_I the steady-state nominal output shares of the two sectors and by λ_K and λ_L the steady-state income shares of capital and labor. Appendix A.1 derives the transition system.

Proposition 1.1 (Long-run elasticities). *The long-run elasticities of the price and quantity of capital with respect to a capital-demand shock are:*

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \underbrace{\frac{\alpha_C-\alpha_I}{1-\alpha_I}}_{\text{technology asymmetry}} + \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_C}}_{\text{labor specialization}},$$

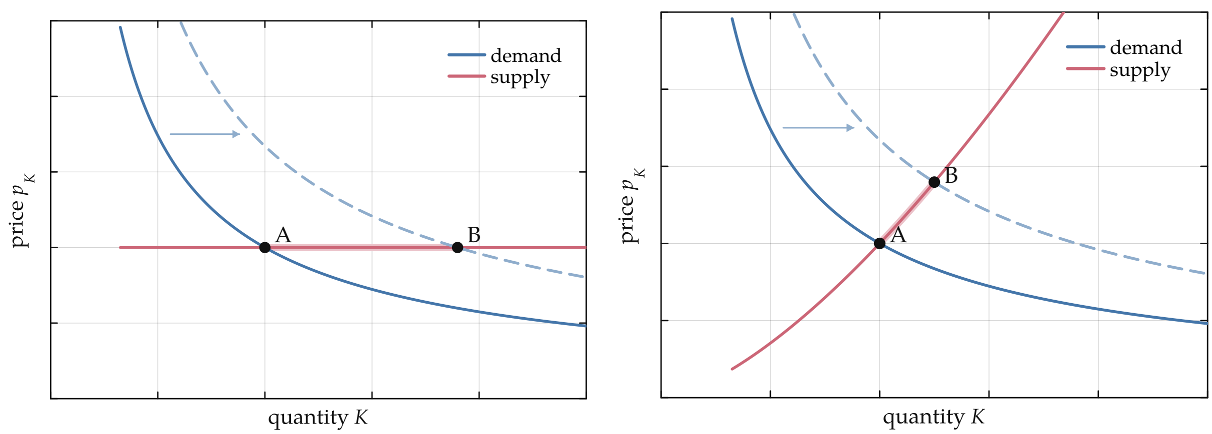
$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{\lambda_L}\left(1-\underbrace{(\alpha_C-\alpha_I)\left(\frac{\lambda_C}{1-\alpha_I}+\frac{\lambda_I}{\alpha_C}\right)}_{\text{technology asymmetry}}-\underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\lambda_K}_{\text{labor specialization}}\right).$$

Appendix A.1 derives Proposition 1.1. In the special case of the one-sector neoclassical growth model ($\alpha_C = \alpha_I$ and $\phi = \infty$), the proposition gives the classical result: the elasticity of the capital price is zero, while the elasticity of the capital stock is the inverse of the labor share.

Outside of this special case, the elasticity of the capital price is the sum of two components. The first reflects *technology asymmetry*, and is positive when $\alpha_C > \alpha_I$, i.e., when the investment sector is more labor-intensive than the consumption sector. Intuitively, if this is the case, anything that makes labor more expensive relative to capital (such as a shift in the demand for capital) raises the cost of producing investment goods relative to consumption goods. This is exactly the same logic as in the Stolper–Samuelson theorem in international trade: the price of a good increases with the price of the factor used more intensively in its production.

The second term reflects *labor specialization*, and is positive as long as ϕ is finite. Intuitively, as soon as $\phi < \infty$, it is costly to reallocate labor from C to I, which creates a bottleneck when the economy tries to respond to a shock to desired saving by producing more capital. This bids up the price of labor in the investment sector relative to the consumption sector, which in turn pushes up the relative price of capital.

The long-run elasticity of capital quantity is the sum of three terms. The first corresponds to the one-sector neoclassical benchmark, while the remaining two are dampening terms with exactly the same interpretations as the two components of the price elasticity.[1]



(a) Neoclassical growth model: $\alpha_C = \alpha_I$ and $\phi = \infty$ (b) Stylized model: $\alpha_C > \alpha_I$ or $\phi < \infty$

Figure 1: Long-run demand shock in the capital market

Notes: The long-run demand curve combines the intratemporal block with capital valuation; the supply curve combines it with capital accumulation. Appendix A.1 gives both conditions.

Figure 1 illustrates the situation graphically, by comparing a shift in the demand for capital in the one-sector neoclassical growth model versus our stylized two-sector model. The more inelastic the capital supply (a steeper supply curve), the more the shock is absorbed through prices rather than quantities. In the neoclassical limit, with perfectly elastic capital supply, demand shocks translate entirely into quantities. In a more realistic model, where the supply of capital is not perfectly elastic, demand shocks are absorbed by a mix of higher quantities and higher capital prices.

[1] Appendix A.1 gives the exact price–quantity mapping.

Response of factor prices. The positive elasticity of the capital price to demand shocks has direct implications for the incidence of saving (capital-tax) shocks. In the one-sector neoclassical growth benchmark, a decrease in the capital tax is fully absorbed by higher capital quantity (instead of higher capital prices): in the long run, the post-tax rental rate of capital remains the same while the wages of workers increase, reflecting the higher marginal product of labor. In our stylized model, however, the supply of capital is less elastic, and so the incidence of lower capital taxes is partly tilted towards capitalists.

To see why, use the fact that, due to unit cost pricing in each sector, the weighted change in output prices in the economy must equal the weighted change in factor prices:

$$\lambda_C \,\mathrm{d}\log p_C + \lambda_I \,\mathrm{d}\log p_K = \lambda_K \,\mathrm{d}\log r_K + \lambda_L \,\mathrm{d}\log w,$$

where $\mathrm{d}\log w$ denotes the change in the aggregate wage. Because the consumption good is the numeraire (i.e., $\mathrm{d}\log p_C = 0$), the left-hand side only shows the change in the price of the investment good. Moreover, in the long run, the valuation equation implies that the relative price of capital moves as much as the after-tax return on capital:

$$\mathrm{d}\log p_K = \mathrm{d}\log(1-\tau) r_K$$

Combining both, we can express labor income growth in terms of the capital-demand shock and the capital price response:

$$\lambda_L \,\mathrm{d}\log w = \lambda_K \,\mathrm{d}\log(1-\tau) - (\lambda_K - \lambda_I) \,\mathrm{d}\log p_K.$$

In the neoclassical benchmark, $\mathrm{d}\log p_K = 0$, so a decrease in taxes does not affect the post-tax compensation of capital holders while wages increase by λ_K/λ_L. Put differently, workers entirely capture the effect of lower taxes through higher wages. However, in our stylized model with $\alpha_C \neq \alpha_I$ and/or $\phi < \infty$, part of the shock is absorbed by p_K, so the pre-tax rental rate falls by *less than* one-for-one and wages rise by *less than* λ_K/λ_L. In incidence terms, some of the shock is borne by capital holders (through higher post-tax capital payments) rather than workers.[2] We will quantify the welfare incidence of capital taxes more precisely in the calibrated model in Section 4.

[2] Appendix A.1 gives the exact incidence identity.

1.2 Extensions

Our stylized model departs from the neoclassical growth benchmark along two axes: (i) heterogeneous factor intensities across the consumption and investment sectors; and (ii) limited sectoral labor mobility. In Appendix A, we augment the stylized model with a number of extensions one by one. We describe below how each extension affects our comparative static of interest.

CES production. The stylized model assumes Cobb–Douglas production. Appendix A.2 derives the linearized equilibrium conditions and long-run elasticities when sectoral production functions are CES. The new sufficient statistics are the sectoral substitution elasticities. When substitution between capital and labor is easier, the quantity response of K is larger, as in the neoclassical benchmark. Nevertheless, factor-intensity heterogeneity ($\alpha_C \neq \alpha_I$) or labor specialization ($\phi < \infty$) continues to dampen the adjustment of K to demand shocks as in the Cobb–Douglas case.

Multiple factors of production. We extend the model with $f \in \{1, \ldots, F\}$ fixed factors L_f (e.g., labor by type, land) and $n \in \{1, \ldots, N\}$ reproducible factors K_n (e.g., structures, equipment, intangibles). This allows the model to account for sectoral differences in the labor mix (e.g., by education or occupation) and in the composition of capital goods. The key insight for us—which we use in the calibration strategy of Section 3—is that even if total labor shares coincide across sectors C and I, differences in the type of labor hired can make the labor supply faced by a sector less elastic for a given sectoral-equivalent elasticity (e.g., the production of R&D disproportionately requires scientists). Appendix A.3 derives the corresponding long-run system.

Dynamic multi-occupation labor supply. The canonical Roy–Fréchet labor supply is perhaps better suited for the type of long-run experiment we have done thus far, rather than for transitions. Appendix A.4 derives a dynamic occupation block that combines occupation mobility and transition speed in one system. Financial discounting remains governed by r, while the occupation-specific payoff from the current choice decays at rate μ because workers can reoptimize. The transition parameter μ makes labor supply less elastic in the short run without changing the long-run occupation-supply equations. We use this setup in the calibration strategy of Section 3.

Internal adjustment costs and sector-specific capital. In the stylized model, households own the capital stock and rent it to firms. In practice, firms own their own capital and face internal resource bottlenecks when trying to scale too fast (engineering time, management attention), which drives a wedge between the value of businesses and the replacement cost of their capital stock. Appendix A.5 extends the model with (i) an internal adjustment cost as in the standard q-theory of investment and (ii) sector-specific capital stocks. The calibration strategy of Section 3 uses external evidence on internal adjustment costs to discipline this force, which is separate from our theory which is based on aggregate resource bottlenecks.

Imported inputs and modified Euler equations. Appendix A.6 adds imported inputs as an Armington CES bundle, which allows trade openness to absorb part of the capital-demand shock through import quantities at a fixed world price. Appendix A.7 combines the spender-saver aggregation block and the discounted Euler equation. These extensions affect the transition and the mapping from capital-demand shocks to real rates, but the long-run supply elasticities continue to be disciplined by the same production and labor-supply objects.

2 Measuring technology asymmetry and labor specialization

We measure two features of capital production in U.S. data. Technology asymmetry is the difference between the factor requirements of investment and consumption. Labor specialization is the extent to which investment and consumption rely on occupation bundles that do not share workers through observed job transitions. Investment is more labor-intensive than consumption, especially for intangible capital, and draws on occupation bundles that remain distinct after accounting for worker transitions.

2.1 Measurement

Technology asymmetry. We combine the BEA Make and Use Tables with the BEA–BLS Integrated Industry-Level Production Account for 1963–2024 (U.S. Bureau of Economic Analysis, 2024a,b). We divide private final demand into consumption and four investment goods: equipment, nonresidential structures, intellectual property products (IPP), and brand/organizational services. The last category capitalizes selected professional and management services that the national accounts treat as intermediate inputs, following the broad treatment of intangible investment in Corrado et al. (2009).

We first use the Make Table and the commodity-technology assumption to map industry factor payments into commodity factor requirements. We then use the Leontief inverse to include the labor and capital embodied in intermediate inputs and aggregate commodities with their expenditure shares in each final use. Labor compensation is divided among high-school, college, and post-college workers. Land is imputed in agriculture and residential real estate. We divide the remaining capital income among equipment, structures, IPP, and organizational capital using acquisition-value stocks and user costs, and allocate each type across final uses with nominal investment shares. Appendix B.2 gives the input–output accounting, Appendix B.4 gives user costs and returns, and Appendix B.1 gives source definitions.

Labor specialization. We divide each sector-by-education labor share into occupation payroll shares. The decennial Census and American Community Survey measure occupation bundles by industry; the CPS ASEC supplies previous-year and current occupations for workers ages 25–54. We pool the 1976–2024 CPS transitions to measure pairwise occupational mobility and combine them with the same total-requirements occupation bundles used in the factor accounting.

Two destination occupations are closer when their workers arrive from similar origin distributions. We normalize these shared-origin profiles and use the resulting pairwise comparisons to relate the consumption and investment occupation bundles. We combine the education-specific comparisons with each group's share of their joint wage bill. The specialization index runs from zero for identical bundles, or for uniformly random transitions, to one when the bundles have no direct or flow-mediated overlap; figures and tables report the index in percent. Higher values therefore mean more specialized labor. A common rescaling of the pairwise-flow matrix leaves the index unchanged. Appendix B.3 gives the construction, and Appendix B.5.2 reports its robustness to occupation aggregation, transition weighting, education pooling, and origin weights.

2.2 Evidence

Table 1 reports factor shares for the private economy, consumption, aggregate investment, and the four investment goods. The aggregate labor share is 48%. The labor share is 45% in consumption and 60% in investment, a gap of 15 percentage points. Most of the gap is skilled labor: the college-or-above share is 16 percentage points higher in investment.

The contrast is sharpest for intangible investment. The labor share is 51% for equipment and structures and 66% for IPP and brand/organizational services. Consumption uses more structures capital, reflecting housing services, while IPP production uses more IPP capital.

Table 1: Factor shares by sector (2020–2024)

	C+I share	Factor shares							
		Labor			Land	Capital			
		HS	College	Post-college		Equipment	Structures	IPP	Brand/Org
$C+I$	1.00	0.20	0.15	0.13	0.03	0.13	0.10	0.11	0.14
C	0.77	0.20	0.13	0.12	0.03	0.14	0.12	0.10	0.15
I	0.23	0.19	0.23	0.18	0.00	0.10	0.03	0.15	0.12
$I_{\text{Equipment}}$	0.06	0.22	0.17	0.11	0.00	0.13	0.04	0.19	0.15
$I_{\text{Structures}}$	0.03	0.35	0.13	0.06	0.00	0.16	0.07	0.06	0.16
I_{IPP}	0.06	0.14	0.27	0.21	0.00	0.07	0.02	0.20	0.09
$I_{\text{Brand/Org}}$	0.08	0.15	0.29	0.25	0.00	0.07	0.03	0.13	0.09

Notes: Entries pool the 2020–2024 annual estimates. Capital-type shares use the user-cost weights defined in Appendix B.4.

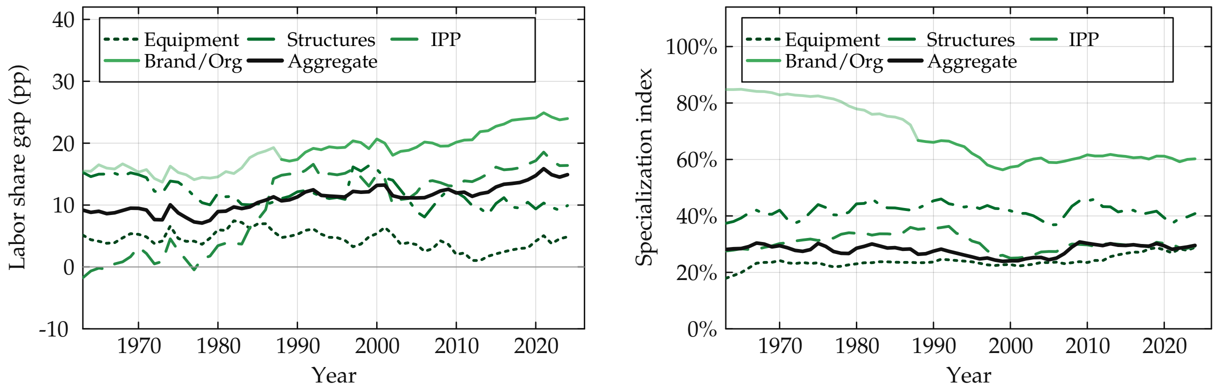
Table 2 shows the occupation bundles behind the specialization measure. Structures concentrate their wage bill in mechanics and construction, while IPP and brand/organizational services rely more heavily on technical and managerial occupations. Consumption has larger health, teaching, and personal-service shares. The transition data determine how much these visibly different bundles nevertheless draw workers from the same origins.

Table 2: Occupation composition of consumption and investment (2020–2024)

Occupation	Consumption	Investment	Equipment	Structures	IPP	Brand/Org
Exec./admin./managerial	0.00	0.13	0.00	0.00	0.00	0.34
Management related	0.09	0.14	0.08	0.07	0.19	0.17
STEM	0.05	0.15	0.18	0.07	0.18	0.13
Science/arts	0.05	0.08	0.03	0.02	0.14	0.08
Doctors/lawyers	0.08	0.02	0.02	0.01	0.03	0.03
Health services	0.09	0.01	0.00	0.00	0.01	0.01
Postsec. teachers	0.01	0.00	0.00	0.00	0.00	0.00
Teachers/librarians	0.05	0.01	0.01	0.01	0.01	0.00
Health/sci. techs.	0.06	0.10	0.11	0.03	0.17	0.08
Sales	0.13	0.09	0.16	0.08	0.09	0.05
Admin. support	0.10	0.07	0.09	0.07	0.07	0.05
Protective services	0.01	0.01	0.01	0.01	0.01	0.01
Food/cleaning/personal	0.07	0.02	0.01	0.01	0.02	0.02
Farm/extraction	0.02	0.01	0.01	0.02	0.01	0.01
Mechanics/construction	0.07	0.08	0.06	0.44	0.02	0.01
Precision mfg.	0.02	0.02	0.05	0.02	0.01	0.01
Mfg. operators	0.03	0.02	0.05	0.03	0.01	0.01
Fabricators/handlers	0.03	0.03	0.09	0.04	0.01	0.01
Vehicle operators	0.04	0.03	0.05	0.07	0.01	0.01

Notes: Entries are occupation shares of each final use's occupation wage bill, pooled over 2020–2024. Investment pools the four component wage bills before normalizing; each column sums to one before rounding. See Appendix B.3.

Figure 2 shows both measures over time. In panel (a), each line subtracts consumption's labor share from the labor share of one investment good; a positive value means that the investment good is more labor-intensive. The black aggregate rises from 9 percentage points in 1963–1968 to 15 percentage points in 2020–2024. In panel (b), higher lines mean that the investment good uses a more specialized occupation bundle relative to consumption.



(a) Labor-share gap

(b) Occupation specialization

Figure 2: Technology asymmetry and labor specialization

Notes: Panel (a) reports labor-share differences from consumption in percentage points; panel (b) reports occupation specialization relative to consumption, from zero (complete overlap) to 100 (no overlap). Both panels use the capital-type shades and line styles in Figure A2; black denotes aggregate investment. Brand/Org is capitalized professional and management services; its lighter pre-1988 segment is a backcast. Appendices B.2 and B.3 give the two constructions.

Appendices B.5.1 and B.5.2 report parallel robustness exercises. The technology checks vary organizational-investment scope, government and import treatment, the commodity mapping, and input–output aggregation. The labor checks vary occupation aggregation, transition weighting, education pooling, and origin weights. The labor-share gap and occupation specialization survive these alternatives. Two choices matter quantitatively: omitting the brand/organizational capitalization lowers the 2020–2024 labor-share gap from 15 to 10 percentage points (and its rise since 1963–1968 from 6 to 3), and excluding housing from consumption lowers the gap from 15 to 8.

3 Quantitative model

We calibrate an extended version of the stylized model from Section 1 and quantify the implied dynamic response to demand shocks. After presenting the transition dynamics, we validate the model's price and quantity predictions. Section 4 then uses the calibrated model to study the incidence of capital taxation.

We introduce $N = 5$ capital types (equipment, structures, IPP, brand/organizational, and land), five sectors (consumption and four investment goods), and three education groups. Land is not produced: it has no investment good, so $\delta_h = 0$ and $\lambda_{I_h} = 0$, and its stock is fixed.

Different demand shocks therefore affect different workers: intangible-capital shocks disproportionately benefit scientists, while structures-investment shocks benefit construction workers. Labor-mobility data are observed by occupation rather than by sector. We therefore take occupations as the model's primitive labor margin and use sectoral-equivalent elasticities only in the interpretive decomposition below.

Production uses factor cells L_{sfo}, where $s \in \{C, I_1, \dots, I_4\}$ is the sector (land, the fifth capital type, has no investment sector), f is the education group, and o is the occupation. The factor share of cell (f, o) in sector s is $\alpha_{sL_{fo}} = \alpha_{sL_f} \gamma_{sfo}$. Worker cells are mobile across sectors within an occupation cell, so they earn a common return w_{fo}; sectoral occupation demands aggregate to the occupation stock L_{fo}. Worker reallocation across occupations is governed by education-specific cross-elasticities $\phi_{foo'}$. The full quantitative model expands each education group into $O = 66$ occupation cells. Throughout the equations, $\sum_{fo}$ runs over worker cells and $\sum_m$ runs over all N capital types, land included.

Table 3: Linearized model equations

Intratemporal block	Equation
C pricing	$0 = \sum_m \alpha_{CK_m} \,\mathrm{d}\log r_{K_{Cm}} + \sum_{fo} \alpha_{CL_{fo}} \,\mathrm{d}\log w_{fo}$
I_n pricing	$\mathrm{d}\log p_{I_n} = \sum_m \alpha_{I_nK_m} \,\mathrm{d}\log r_{K_{I_nm}} + \sum_{fo} \alpha_{I_nL_{fo}} \,\mathrm{d}\log w_{fo}$
K_{sn} demand	$\mathrm{d}\log K_{sn} = \mathrm{d}\log Y_s + \sum_{m \neq n} \alpha_{sK_m} \,\mathrm{d}\log \frac{r_{K_{sm}}}{r_{K_{sn}}} + \sum_{fo} \alpha_{sL_{fo}} \,\mathrm{d}\log \frac{w_{fo}}{r_{K_{sn}}}$
L_{sfo} demand	$\mathrm{d}\log L_{sfo} = \mathrm{d}\log Y_s + \sum_m \alpha_{sK_m} \,\mathrm{d}\log \frac{r_{K_{sm}}}{w_{fo}} + \sum_{f'o' \neq fo} \alpha_{sL_{f'o'}} \,\mathrm{d}\log \frac{w_{f'o'}}{w_{fo}}$
L_{fo} clearing	$\mathrm{d}\log L_{fo} = \sum_s \frac{\lambda_s \alpha_{sL_{fo}}}{\lambda_{L_{fo}}} \,\mathrm{d}\log L_{sfo}$
Intertemporal block	**Equation**
K_{sn} valuation	$\mathrm{d}\log((1 + r_{t+1}) p_{K_{sn},t}) = \frac{\lambda_{K_{sn}}}{\lambda_{W_{sn}}} \,\mathrm{d}\log((1 - \tau_{n,t+1}) r_{K_{sn},t+1}) - \frac{\lambda_{I_{sn}}}{\lambda_{W_{sn}}} \,\mathrm{d}\log p_{I_n,t+1} + \frac{1}{1+r} \,\mathrm{d}\log p_{K_{sn},t+1}$
K_{sn} accumulation	$\mathrm{d}\log K_{sn,t+1} = (1 - (1 - \theta_n)\delta_n) \,\mathrm{d}\log K_{sn,t} + (1 - \theta_n)\delta_n \,\mathrm{d}\log I_{sn,t}$
I_{sn} investment-q	$\mathrm{d}\log I_{sn,t} = \mathrm{d}\log K_{sn,t} + \frac{1}{\theta_n} \,\mathrm{d}\log \frac{p_{K_{sn},t}}{p_{I_n,t}}$
C Euler equation	$\mathrm{d}\log C_t = \mathrm{d}\log C_{t+1} - \psi \,\mathrm{d}\log(1 + r_{t+1})$
L_{fo} valuation	$\mathrm{d}\log\Big((1 + r_{t+1}) p_{L_{fo},t}\Big) = \frac{r+\mu_f}{1+r} \,\mathrm{d}\log w_{fo,t+1} + \frac{1-\mu_f}{1+r} \,\mathrm{d}\log p_{L_{fo},t+1}$
L_f accumulation	$\mathrm{d}\log L_{f,t+1} = (1 - \nu) \,\mathrm{d}\log L_{f,t} + \nu\eta \,\mathrm{d}\log \frac{p_{L_f,t}}{p_{L,t}}$
L_{fo} accumulation	$\mathrm{d}\log L_{fo,t+1} = \mathrm{d}\log L_{f,t+1} + (1 - \mu_f) \,\mathrm{d}\log \frac{L_{fo,t}}{L_{f,t}} + \mu_f \sum_{o'} \phi_{foo'} \,\mathrm{d}\log \frac{p_{L_{fo'},t}}{p_{L_f,t}}$

Notes: $\mathrm{d}\log x$ denotes a log-deviation from steady state, Y_s is sector s's output (C or I_n), and λ denotes an income share. Land is the non-produced capital type h: $\delta_h = 0$ makes its accumulation equation hold its stock fixed, and with no investment good it has no investment-q condition. Appendix A.5 defines the sector-type shares $\lambda_{W_{sn}}$, $\lambda_{I_{sn}}$, and $\lambda_{K_{sn}}$. The value indexes are $\mathrm{d}\log p_{L_f,t} = \sum_o (\lambda_{L_{fo}} / \lambda_{L_f}) \,\mathrm{d}\log p_{L_{fo},t}$ and $\mathrm{d}\log p_{L,t} = \sum_f \left(\lambda_{L_f} / \sum_{f'} \lambda_{L_{f'}}\right) \mathrm{d}\log p_{L_f,t}$. The system is closed by goods-market clearing, $\mathrm{d}\log Y_C = \mathrm{d}\log C$ and $\mathrm{d}\log Y_{I_n} = \mathrm{d}\log I_n$ with $\mathrm{d}\log I_{n,t} = \sum_s (\lambda_{I_{sn}} / \lambda_{I_n}) \,\mathrm{d}\log I_{sn,t}$, omitted from the table for compactness. The L_f accumulation equation is a postulated reduced-form cohort block: a share ν of workers is replaced each year and new cohorts choose education with elasticity η; its long run is $\mathrm{d}\log L_f = \eta \,\mathrm{d}\log(p_{L_f} / p_L)$.

To simulate transition dynamics, we augment the model with a dynamic labor-supply block where cohorts sort into education groups and then across occupations in response to relative wages. Labor specialization is induced by sectors' different occupation bundles. We also include sector-specific capital stocks subject to internal adjustment costs, aimed at matching firm-level evidence on sluggish investment responses to Tobin's q.

Linearized equations. We focus directly on the linearized model around the steady state in response to an arbitrary sequence of type-specific demand shocks $\left((\mathrm{d}\log(1-\tau_{n,t}))_{n=1}^{N}\right)_{t=0}^{\infty}$. As in the stylized model, this shock maps directly to a capital income tax shock, but is also isomorphic to Euler equation shocks. Since the model is a direct extension of the stylized model from Section 1, we list the linearized model equations in Table 3, and refer the reader to Appendix A.3, Appendix A.4, and Appendix A.5 for formal derivations of the extensions. The new parameters of the labor block will be introduced formally in the calibration section below.

The equations are broken down into the intratemporal block (which determines the allocation given the state of the economy) and the intertemporal block (which pins down the evolution of the state). The states are the stocks of capital and labor, and the co-states are the associated present values of future income flows from these factors. Table 3 keeps the production side compact; the quantitative system expands the labor equations to the occupation cells described above.

3.1 Calibration

Demand side. First, we calibrate the demand side of the model: a real annual growth-adjusted required return, $r = 0.0378$, and a unit intertemporal elasticity of substitution, $\psi = 1$. The required return is the expected return on the unlevered business sector claim minus trend output growth, $r = \mathbb{E}r - \bar{g} = 0.0598 - 0.0220$. Appendix B.4 gives the construction and the growth adjustment.

Supply side. We average the production-side inputs over 2020–2024. The sectoral factor shares, α, and income shares, λ, come from BEA IO/KLEMS. Within sector s and education group f, occupation o's wage-bill share, γ_{sfo}, comes from Census/ACS data passed through the same input–output accounting. The annual depreciation rate for capital type n, δ_n, comes from the BEA Fixed Asset Accounts; organizational capital uses the IPP rate. Table 1 reports the main factor shares. Appendix B.2 reports the annual source moments and the inputs used in the model; Appendix B.4 gives user costs and returns.

The annual cohort replacement rate, ν, and education-choice elasticity, η, come from Adão et al. (2024, Table 2). For each education group, we estimate the annual occupation-reoptimization rate, μ_f, from one-year CPS occupation transitions. We construct the long-run occupation labor-supply cross-elasticities, $\phi_{foo'}$, from the same pairwise flows following Böhm et al. (2025) and scale them to Hsieh et al. (2019, Table II). Appendix A.4 gives the occupation-mobility model, and Appendix B.3 gives the empirical construction.

Capital adjustment. We calibrate θ_n separately by capital type using each type's BEA depreciation rate and firm-level evidence on the response of investment to Tobin's q. Table 4 summarizes the baseline parameters and sources. Appendix A.5 gives the model, and Appendix B.4 defines the empirical estimand and maps it to θ_n.

Table 4: Baseline calibration

Parameter	Value	Source
Required return, r	0.0378	IMA/BEA value and payouts
Intertemporal elasticity, ψ	1	–
Sectoral factor shares, (α, λ)	Table A4	BEA IO/KLEMS, 2020–2024
Sectoral occupation shares, γ	Table 2	Census/ACS, 2020–2024
Depreciation, δ	(0.14, 0.03, 0.25, 0.25)	BEA Fixed Assets
Cohort-level education, (ν, η)	(0.03, 0.4)	Adão et al. (2024, Table 2)
Occupation reoptimization, μ	(0.08, 0.07, 0.06)	CPS ASEC transitions, 1976–2024
Occupation labor-supply elasticities, ϕ	Appendix B.3	CPS flows; Hsieh et al. (2019, Table II)
Adjustment costs, θ	(0.20, 0.07, 0.29, 0.29)	Cummins et al. (1996)

Notes: Dimensions are sector by factor for α and λ, sector by education by occupation for γ, capital type for δ and θ, education for μ, and education by origin occupation by destination occupation for ϕ. Capital types are ordered equipment, structures, IPP, and brand/organizational; education groups are ordered high-school, college, and post-college.

3.2 How elastic is the supply of capital?

We start by quantifying the response of capital and valuations to a permanent demand shock $\mathrm{d}\log(1-\tau) = 1_{t\geq 0}$ and focus on aggregate quantities and prices.[3] Throughout this section, we will refer to Table 5, which summarizes the term structure of these responses in the calibrated model (e.g., on impact, at a planning horizon of 10 years, and in the long run). To tease out which of the two forces emphasized in Section 1 matters more quantitatively, we calibrate two alternative models where we add one ingredient at a time to the neoclassical growth model. Similarly, we consider a model extension with only capital adjustment costs (i.e., Hayashi's q-theory of investment).

[3] Aggregate capital quantities and prices use steady-state capital-income weights λ_{K_n}/λ_K.

Table 5: Summary of price and quantity responses (permanent demand shock)

	On impact	Medium-run		Long-run	
	$\mathrm{d}\log p_{K,0}$	$\mathrm{d}\log p_{K,10}$	$\mathrm{d}\log K_{10}$	$\mathrm{d}\log p_{K,\infty}$	$\mathrm{d}\log K_{\infty}$
Calibrated model	1.22	0.37	0.80	0.30	1.39
Neoclassical growth model	0.00	0.00	1.63	0.00	1.89
+capital adjustment costs	0.32	0.06	1.55	0.00	1.89
+technology asymmetry	0.02	0.15	1.40	0.17	1.57
+labor specialization	0.94	0.21	1.20	0.11	1.77
Demand-side sensitivities					
intertemporal elasticity $\psi = 0.5$	+14	−4	−8	0	0
intertemporal elasticity $\psi = 2$	−9	+3	+5	0	0
spender-saver $\omega = 0.25$	+4	0	0	0	0
spender-saver $\omega = 0.50$	+10	−1	−1	0	0
discounted Euler $\varrho = 0.98$	−3	0	−1	+1	+4
discounted Euler $\varrho = 0.95$	−8	0	−2	+3	+10
Supply-side extensions					
imported inputs $\sigma_M = 3$	−12	−4	+24	−5	+51
imported inputs $\sigma_M = 5$	−7	−3	+34	−4	+84
capital–labor elasticity $\sigma_{KL} = 0.5$	−21	−10	−28	−6	−65
capital–labor elasticity $\sigma_{KL} = 1.5$	+13	+8	+20	+5	+59
occupation transition speeds $2 \times \mu_f$	0	−1	+1	0	0
occupation transition speeds $10 \times \mu_f$	−1	−3	+3	0	0
occupation elasticities $2 \times \phi_{foo'}$	0	−2	+1	−2	+3
occupation elasticities $0.5 \times \phi_{foo'}$	0	+2	−1	+2	−2
Alternative data construction					
no capitalization of org. services	−8	−6	0	−8	+3
capitalization of financial services	+5	0	+3	−1	+11
consolidate government sector	−1	−3	+3	−4	+2
common industry-tech. assumption	+3	−2	+4	−4	+10

Notes: The first panel reports price shares of the demand shock and quantity log changes. The remaining panels report signed percentage-point deviations from the calibrated model (100× the difference).

The sensitivity panels of Table 5 report alternative modified Euler equations, trade openness, production substitution, and labor mobility. First, we vary the elasticity of intertemporal substitution $\psi \in \{0.5, 2\}$, bracketing the baseline $\psi = 1$ (log utility). Second, we use the joint spender-saver discounted-Euler block of Appendix A.7. We report spender-saver rows with hand-to-mouth agents consuming an ω share of fixed-factor income, including land, while savers own reproducible capital and follow the Euler equation. We also report modified Euler equations with attenuation parameter $\varrho \in \{0.98, 0.95\}$, which dampen the effect of distant real rates and can move the long-run real rate. The supply-side extensions introduce imports as a CES composite with domestic factors, with Armington trade elasticity $\sigma_M \in \{3, 5\}$ and sector-specific import shares from BEA domestic input–output tables (see Table A9). They also replace Cobb–Douglas production with CES production, using a common capital–labor substitution elasticity $\sigma_{KL} \in \{0.5, 1.5\}$ in every sector (Appendix A.2). Finally, they scale the occupation cross-elasticities by two and by one half, which changes the long-run capital-price response by -2 and +2 percentage points, respectively.

The calibrated quantity response is 0.80 after ten years and 1.39 in the long run, compared with 1.63 and 1.89 in the neoclassical growth model. Thus benchmark calculations based on the one-sector model materially overstate the effect of a demand-side policy on capital accumulation.

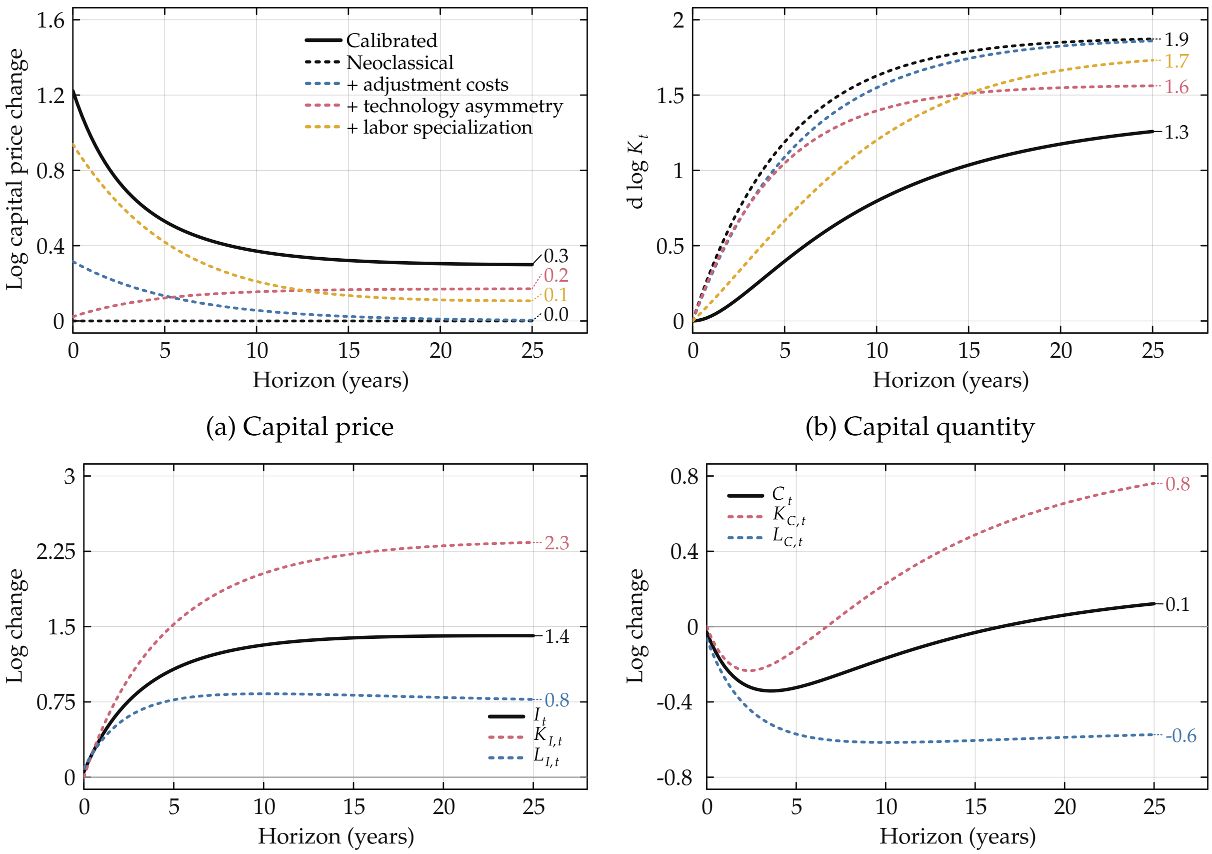
The weak quantity response has a price counterpart. In the calibrated model, capital-good prices absorb 0.37 of the shock after ten years and 0.30 in the long run, whereas their long-run response is zero in the neoclassical model. The impact revaluation is 1.22 and decays toward its long-run level over roughly a decade.

Long-run decomposition. We next decompose the long-run price effect into its fundamental drivers. Conditional on the steady-state valuation shares $\lambda_{K_{sn}}$, $\lambda_{I_{sn}}$, and $\lambda_{W_{sn}}$, the long run is invariant to capital adjustment costs θ and intertemporal substitution ψ. Recomputing those shares after changing θ instead changes the steady-state linearization point. Holding that point fixed lets us decompose the long-run price effect into technology asymmetry ($\alpha_C > \alpha_I$), labor specialization, and capital-type heterogeneity.

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \underbrace{\frac{\alpha_{CK}-\alpha_{IK}}{1-\alpha_{IK}}}_{\substack{\text{technology asymmetry}\\ =0.17}} + \underbrace{\frac{1}{1+\sum_f \frac{\lambda_{L_f}}{\lambda_L}\phi_f}\,\frac{1-\alpha_{CK}}{\alpha_{CK}}\,\frac{\lambda_K}{\lambda_C}}_{\substack{\text{labor specialization}\\ =0.09}} + \underbrace{\mathrm{Cov}_L}_{=0.03} + \underbrace{\mathrm{Cov}_K}_{=0.01}.$$

The first two terms are the sufficient-statistic benchmarks from Proposition 1.1, evaluated using aggregate capital shares and the labor-income-weighted average of the implied sectoral-equivalent elasticities ϕ_f, defined in Appendix A.3. The remaining terms restore the calibrated system in sequence. Cov_L replaces the scalar labor elasticity with the education-by-occupation supply block and fixed land in a one-capital aggregation. Cov_K then restores the four capital types, the network of investment sectors, their calibrated valuation equations, and the interaction between capital composition and occupation demand. Appendix A.3 derives the reduced long-run system. Each Cov term is defined as the exact difference between two nested long-run systems, so the four displayed terms add to the calibrated response of 0.30.

Transition dynamics. Figure 3 plots the transition paths of prices and quantities. The price response is much larger on impact than in the long run: 1.22 versus 0.30. The benchmark models separate the contributions of adjustment costs, labor specialization, and technology asymmetry to this overshoot.



(a) Capital price

(b) Capital quantity

(c) Investment sector

(d) Consumption sector

Figure 3: Capital-market transition dynamics and input reallocation

Notes: Panels (a)–(b) report the variants defined in Table 5. Panels (c)–(d) use factor-share weights.

Capital adjustment costs and labor specialization matter disproportionately in the short run. Conditional on the steady-state valuation shares, adjustment costs have no independent long-run implication. Labor supply adjusts gradually at rates governed by $\mu_f, \nu \in (0,1)$, while the technology-asymmetry force builds with capital accumulation. Table 5 reports the responses in each special case.

The gap between the calibrated and neoclassical capital stocks persists throughout the transition. Adjustment costs and labor specialization drive most of the short-run gap, while technology asymmetry matters more in the long run. Panels (c)–(d) of Figure 3 show the underlying reallocation. Capital stocks are predetermined on impact, while labor immediately moves from consumption toward investment, raising investment output and temporarily lowering consumption. As capital accumulates, capital services in consumption rise; specialized labor nevertheless remains tilted toward investment even in the long run. Consumption crosses its initial level after roughly 16 years, and both sectors ultimately expand.

Validation. We benchmark the model against existing evidence on the transmission of tax shocks to real investment as well as relative prices. Importantly, we need empirical designs that recover the aggregate effect of the shocks (which work via general equilibrium price changes), not designs that only recover relative effects across firms, goods, etc.

The evidence is limited to the very short run and mostly covers investment in equipment goods. Table 6 organizes and harmonizes the results by reporting the division of investment expenditure responses into quantities and prices (or between hours and wages when looking at sectoral labor markets), both in the data and the calibrated model. Overall, the literature finds a modestly inelastic short-run supply of capital goods, with large differences in effect sizes across studies and types of capital goods.

Using 81 equipment industries, Goolsbee (1998b) estimates how equipment output and output prices respond to changes in investment tax incentives. Using CPS data, Goolsbee (1998a) estimates how the hours and wages of scientists and engineers respond to changes in federal R&D spending. House and Shapiro (2008) estimate how real investment and the total cost of investment respond across equipment and structures to temporary bonus depreciation. House, Mocanu and Shapiro (2017) estimate how purchases, production prices, production-worker hours, and wage bills respond across equipment and structures to investment tax subsidies.

Goolsbee (1998b) estimates an equipment supply curve on 81 equipment-producing industries, instrumenting the output price with investment tax terms and without year effects, and the model announces a permanent equipment-demand subsidy a year ahead, so output prices take 0.47 of the nominal response in the data and 0.55 in the model. Goolsbee (1998a) regresses the income of scientists and engineers on federally funded R&D as a share of GDP, an aggregate annual series, and the model announces a permanent IPP-demand subsidy a year ahead, so wages take 0.96 of the combined wage and hours response in the data and 0.98 in the model. House and Shapiro (2008) estimate the response of investment to temporary bonus depreciation across 36 asset types, identified primarily from cross-sectional variation in the incentive, and the model applies the anticipated statutory bonus sequence to equipment, so total investment cost takes 0.12 of the nominal response in the data and 0.85 in the model. House, Mocanu and Shapiro (2017) estimate responses to investment tax subsidies across 28 equipment and 17 structures types without time effects, and the model announces a permanent subsidy to the corresponding investment sector a year ahead, so producer prices take -0.07 of the equipment expenditure response in the data and 0.55 in the model, wages take 0.40 of the equipment payroll response and 0.24, and structures prices take 0.38 and 0.53.

Appendix C.1 reports the underlying coefficients and empirical designs.

Table 6: Price and wage shares of nominal responses

Paper	Sector	Sample	Quantity	Price or wage	Data	Model
Goolsbee (1998b)	Equip.	1959–1988	Output	Output price	0.47 (0.11)	0.55
Goolsbee (1998a)	IPP	1968–1994	Hours	Wage	0.96 (0.06)	0.98
House and Shapiro (2008)	Equip./struct.	2001–2006	Investment	Total cost	0.12 (0.02)	0.85
House, Mocanu and Shapiro (2017)	Equip.	1959–2009	Purchases	Producer price	-0.07 (0.07)	0.55
			Hours	Wage	0.40 (0.78)	0.24
	Struct.		Purchases	Producer price	0.38 (0.14)	0.53

Notes: Data and Model report the price or wage share of the corresponding nominal response; quantity or hours account for the balance. Negative values indicate opposite-signed price and quantity responses. Parentheses report delta-method standard errors. Where a share combines two published coefficients, none of the studies reports their covariance, and we take the largest standard error consistent with any correlation between them. Appendix C.1 gives the constructions.

4 Incidence of capital taxation

We evaluate a permanent capital income tax cut around a steady state with a pre-existing uniform rate $\tau \geq 0$ on reproducible capital income and land rents. At $\tau = 0$ the reform is a transfer from the government to households; at $\tau > 0$ it also raises efficiency. Let $R = 1 + r$ denote the gross return per model period. We normalize the common permanent infinitesimal reform to $\mathrm{d}\log(1 - \tau_{n,t}) = 1$ for every capital type n, land included, and all $t \geq 0$. This is a derivative normalization, so every reported effect scales linearly with the reform. The household's willingness to pay is

$$\mathcal{W} = \sum_{t \geq 1} R^{-t} \left(\sum_n ((1-\tau)\lambda_{K_n} \mathrm{d}\log((1-\tau_{n,t}) r_{K_n,t}) - \lambda_{I_n} \mathrm{d}\log p_{I_n,t}) + \sum_{fo} \lambda_{L_{fo}} \mathrm{d}\log w_{fo,t} \right).$$

Owners receive the after-tax rental of every capital type and the wage bill of every worker cell. The mechanical tax benefit is the $\mathrm{d}\log(1 - \tau_{n,t})$ component of the first term, so under the normalization above it contributes $(1-\tau)\lambda_K/r$; writing it inside the discounted sum keeps the expression valid for tax paths that are not constant. The factors $1 - \tau$ convert gross factor income into the after-tax income received by owners at the distorted steady state.

The government welfare effect from the same reform is

$$\mathcal{G} = -(1-\tau)\frac{\lambda_K}{r} + \tau \sum_{t \geq 1} R^{-t} \sum_n \lambda_{K_n} \mathrm{d}\log\left(r_{K_n,t} K_{n,t}\right).$$

The first term is the mechanical revenue loss and the second is the change in revenue from the taxed income base. Combining household and government effects gives

$$\mathcal{W} + \mathcal{G} = \tau\,\lambda_K \sum_{t\geq 1} R^{-t}\,\mathrm{d}\log K_t. \tag{4.1}$$

Factor-price changes cancel by the linearized unit-cost conditions, and fixed land has no quantity response. Government revenue is valued one-for-one with consumption—equivalently, it is rebated lump sum—so transfers between households and the government cancel in aggregate. At $\tau = 0$, the right-hand side is zero and the tax cut is purely redistributive.

The more inelastic aggregate capital is (i.e., the larger the revaluation effect), the more capitalists benefit from the tax cut. Holding discount rates fixed (for instance in a small open economy), capitalists' welfare gain would equal the impact rise in the value of their existing capital stock. When discount rates respond to the tax cut, a wedge opens between the revaluation and the welfare effect (Fagereng et al., 2025), but inelastic capital supply still shifts incidence toward capitalists relative to the neoclassical benchmark. The efficiency gain as the pre-existing tax τ varies is quantified in Figure 4.

Tables 7–9 report aggregate welfare incidence, wage changes by occupation, and capitalist incidence by capital type.

Results. Table 7 reports welfare incidence at $\tau = 0$ and 0.20, as a share of the net government revenue loss at each tax rate. At $\tau = 0.20$, the calibrated model implies worker and capitalist gains of 1.01 and 0.26, compared with 1.19 and 0.28 in the neoclassical benchmark. Inelastic capital thus lowers worker gains while leaving capitalist gains roughly unchanged at a distorted steady state (a capitalist gap of -2 pp). At $\tau = 0$, where the reform is purely redistributive, capitalists capture 0.29 of the fiscal cost against 0.22 in the neoclassical benchmark. Because the tax cut raises efficiency when $\tau > 0$, household gains sum to more than one when divided by the net fiscal cost; the final column nets out the government loss and reports the aggregate Harberger efficiency term. Among capitalists at $\tau = 0.20$, tangible capital owners capture 0.15 , intangible capital owners 0.06 , and landowners 0.05 . Appendix C.2 reports robustness at $\tau = 0.20$.

Optimal non-uniform capital taxation. Around a distorted steady state, the marginal efficiency gain from a permanent unit-log tax cut is given by (4.1): the tax wedge times the induced aggregate quantity response. To second order around $\tau = 0$, eliminating a tax at rate τ changes $\log(1-\tau)$ by approximately τ. Holding the local capital response fixed and integrating the

Table 7: Welfare incidence of capital income tax cut

	Workers				Capitalists				Government	Aggregate
	Total	HS	Col.	Post-col.	Total	Tang.	Intan.	Land	Total	Total
Panel A: $\tau = 0$										
Calibrated model	0.71	0.27	0.25	0.19	0.29	0.15	0.10	0.05	-1.00	0.00
Neoclassical growth model	0.78	0.34	0.24	0.20	0.22	0.08	0.07	0.08	-1.00	0.00
+capital adjustment costs	0.76	0.33	0.24	0.19	0.24	0.09	0.08	0.08	-1.00	0.00
+technology asymmetry	0.79	0.34	0.25	0.20	0.21	0.07	0.07	0.08	-1.00	0.00
+labor specialization	0.74	0.32	0.23	0.19	0.26	0.10	0.09	0.08	-1.00	0.00
Panel B: $\tau = 0.20$										
Calibrated model	1.01	0.38	0.35	0.27	0.26	0.15	0.06	0.05	-1.00	0.26
Neoclassical growth model	1.19	0.52	0.37	0.30	0.28	0.09	0.09	0.09	-1.00	0.47
+capital adjustment costs	1.16	0.50	0.36	0.29	0.30	0.10	0.10	0.09	-1.00	0.46
+technology asymmetry	1.16	0.50	0.36	0.29	0.22	0.07	0.06	0.09	-1.00	0.38
+labor specialization	1.13	0.49	0.35	0.29	0.26	0.09	0.08	0.09	-1.00	0.39

Notes: Entries are divided by the net government revenue loss, so Government is -1 and Aggregate sums workers, capitalists, and government. Single-factor benchmarks allocate within-group incidence using baseline income shares.

Table 8: Wage incidence by broad occupation

Occupation	Log wage change (annuity value)	Occupation	Log wage change (annuity value)
Executives and Managers	1.04	Administrative Support	0.60
Engineering and Computing	1.00	Sales, All	0.58
Health and Science Technicians	0.85	Agriculture, Logging, Extraction	0.57
Management Related	0.80	Fire, Police, and Guards	0.54
Scientists	0.79	Teachers, Postsecondary	0.48
Mechanics and Construction	0.73	Doctors and Lawyers	0.48
Precision Manufacturing	0.68	Food, Cleaning, Personal Services	0.44
Fabricators and Material Handlers	0.68	Teachers and Librarians	0.39
Manufacturing Operators	0.65	Nurses and Health Services	0.21
Vehicle Operators	0.61		
Aggregate			0.68

Notes: Annuity values equal $\frac{r}{1+r}\sum_{t\geq 0} R^{-t}\,\mathrm{d}\log w_{o,t}$. Education and occupation aggregates use baseline wage-bill weights; the 66 occupations are grouped into the broad categories of Hsieh et al. (2019).

Table 9: Cash flow incidence by capital type

Capital type	After-tax return on capital (annuity value)	Investment good prices (annuity value)	Cash flow (annuity value)
Land	1.03	–	1.03
Structures	0.50	0.25	0.36
Equipment	0.42	0.21	0.25
IPP	0.60	0.42	0.24
Brand/Org	0.67	0.54	0.20
Aggregate	0.57	0.27	0.29

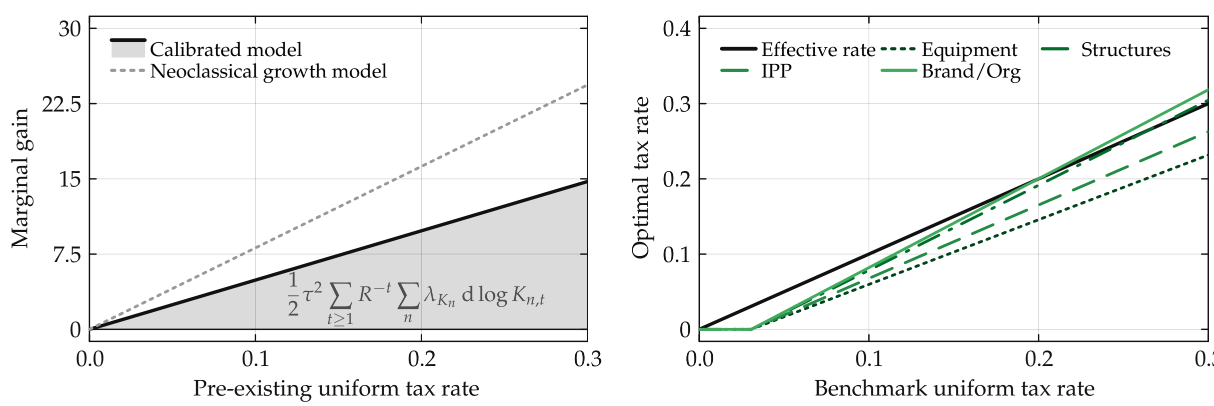
Notes: Evaluated at $\tau = 0$. Annuity values multiply discounted sums by $r/(1+r)$. The columns report $\mathrm{d}\log((1-\tau_n)r_{K_n})$, $\mathrm{d}\log p_{I_n}$, and $\mathrm{d}\log((1-\tau_n)r_{K_n}) - (\lambda_{I_n}/\lambda_{K_n})\,\mathrm{d}\log p_{I_n}$; the first and third include the unit tax change. Aggregate uses baseline income shares.

marginal gain gives the Harberger triangle formula:

$$\frac{1}{2}\tau^2\lambda_K\sum_{t\geq 1}R^{-t}\,\mathrm{d}\log K_t.$$

Figure 4 plots the marginal gain; the shaded area approximates the welfare gain from eliminating the tax.

We choose constant type-specific rates that match the government-welfare effect of the uniform benchmark. The Henry George theorem holds initially: land is in fixed supply, so taxing land rents is nondistortionary and every reproducible-capital rate stays at zero until the land-rent rate reaches one (George, 1879; Arnott and Stiglitz, 1979). But land rents in the modern U.S. economy are too small for this to go far: taxing land rents at 100% replaces a uniform capital income tax of only 3.06%. Above that benchmark, the additional tax burden is shared across reproducible capital types according to how elastic each is, taking cross-type effects into account: every interior rate equates its share of the marginal efficiency cost with its share of the fiscal effect.[4] At every positive benchmark, all four rates on reproducible capital are positive, with Structures > Brand/Org > IPP > Equipment. At $\bar{\tau} = 0.20$, the locally optimal schedule reduces the Harberger triangle by 24.63%; Appendix Table A13 reports the schedules and reductions from $\bar{\tau} = 0$ through 0.50 in increments of 0.10. Appendix C.3 reports incidence at selected benchmarks.



(a) Marginal efficiency gain

(b) Optimal tax rates by type

Figure 4: Efficiency gains and optimal non-uniform taxation

Notes: Panel (a) shades the local Harberger triangle. In panel (b), the black line is the fiscal-weighted effective rate; land is omitted. Appendix C.3 gives the construction.

[4] Appendix C.3 gives the first-order condition and construction.

5 Conclusion

How does the economy absorb a rise in desired saving? The standard growth model says entirely through capital formation. We show that investment in the U.S. economy is more labor-intensive than consumption and draws disproportionately on specialized occupations. As a result, aggregate market clearing implies that scaling up capital production raises the relative price of these scarce inputs, making capital supply upward-sloping even in the long run.

We build a quantitative model disciplined by BEA input–output and KLEMS accounts, combined with Census and CPS data on occupation transitions. The calibrated model places the U.S. economy between the neoclassical benchmark and an endowment economy where aggregate capital is fixed. For instance, 30% of a demand shock is absorbed by prices in the long run, and the 10-year capital-stock response is halved relative to a fully elastic benchmark.

Technology asymmetries and labor specialization have meaningful effects on the macroeconomic transmission of capital-demand shocks. Our analysis also points to unequal incidence across workers and asset owners, which we illustrate by simulating a permanent capital-income-tax cut. Inelastic capital shifts incidence away from workers as a whole and toward asset owners, especially owners of land and structures. Among workers, executives, engineers, and scientists gain the most because their skills become a bottleneck in capital formation.

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Appendix

A Appendix for Section 1

A.1 Stylized model: long-run and transition dynamics

Intratemporal block. The stylized model has one reproducible factor K, one labor aggregate L, and two sectors, C and I. We linearize around a steady state with $\tau = 0$. The intratemporal block is

$$
\begin{aligned}
0 &= \alpha_C \,\mathrm{d}\log r_K + (1-\alpha_C)\,\mathrm{d}\log w_C,\\
\mathrm{d}\log p_K &= \alpha_I \,\mathrm{d}\log r_K + (1-\alpha_I)\,\mathrm{d}\log w_I,\\
\mathrm{d}\log(r_K K) &= \frac{\alpha_C\lambda_C}{\lambda_K}\,\mathrm{d}\log C + \frac{\alpha_I\lambda_I}{\lambda_K}\,\mathrm{d}\log(p_K I),\\
\mathrm{d}\log C &= \phi\,\mathrm{d}\log\left(\frac{w_C}{w}\right) + \mathrm{d}\log w_C,\\
\mathrm{d}\log(p_K I) &= \phi\,\mathrm{d}\log\left(\frac{w_I}{w}\right) + \mathrm{d}\log w_I,\\
\mathrm{d}\log w &= \frac{(1-\alpha_C)\lambda_C}{\lambda_L}\,\mathrm{d}\log w_C + \frac{(1-\alpha_I)\lambda_I}{\lambda_L}\,\mathrm{d}\log w_I.
\end{aligned}
$$

Given $\mathrm{d}\log p_K$ and $\mathrm{d}\log K$, this system determines $\mathrm{d}\log r_K$, $\mathrm{d}\log w_C$, $\mathrm{d}\log w_I$, $\mathrm{d}\log w$, $\mathrm{d}\log C$, and $\mathrm{d}\log I$. Solving gives

$$
\begin{aligned}
\mathrm{d}\log r_K = {} & \frac{(1-\alpha_C)\lambda_I\left(\alpha_I\lambda_L + \phi(\alpha_I-\alpha_C)\lambda_C\right)}{\alpha_C(1-\alpha_I)\lambda_C\lambda_L + \alpha_I(1-\alpha_C)\lambda_I\lambda_L + \phi\lambda_I\lambda_C(\alpha_I-\alpha_C)^2}\,\mathrm{d}\log p_K\\
& - \frac{(1-\alpha_C)(1-\alpha_I)\lambda_K\lambda_L}{\alpha_C(1-\alpha_I)\lambda_C\lambda_L + \alpha_I(1-\alpha_C)\lambda_I\lambda_L + \phi\lambda_I\lambda_C(\alpha_I-\alpha_C)^2}\,\mathrm{d}\log K
\end{aligned}
\tag{A.1}
$$

and

$$
\mathrm{d}\log I = -\frac{\alpha_I}{\alpha_I-\alpha_C}\,\mathrm{d}\log p_K + \frac{(1-\alpha_C)\lambda_K}{(\alpha_I-\alpha_C)\lambda_I}\,\mathrm{d}\log K + \frac{\lambda_K}{(\alpha_I-\alpha_C)\lambda_I}\,\mathrm{d}\log r_K. \tag{A.2}
$$

Equation (A.2) assumes $\alpha_C \neq \alpha_I$; the case $\alpha_C = \alpha_I$ obtains as a finite limit, and all subsequent formulas remain regular there. Substituting (A.1) into (A.2), write

$$
\mathrm{d}\log r_K = \varepsilon_{r,p}\,\mathrm{d}\log p_K + \varepsilon_{r,K}\,\mathrm{d}\log K, \qquad \mathrm{d}\log I = \varepsilon_{I,p}\,\mathrm{d}\log p_K + \varepsilon_{I,K}\,\mathrm{d}\log K.
$$

Long-run elasticities; Proposition 1.1 restated. The long-run elasticities of the price and quantity of capital with respect to a capital-demand shock are:

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \underbrace{\frac{\alpha_C-\alpha_I}{1-\alpha_I}}_{\text{technology asymmetry}} + \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_C}}_{\text{labor specialization}},$$

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{\lambda_L}\left(1-\underbrace{(\alpha_C-\alpha_I)\left(\frac{\lambda_C}{1-\alpha_I}+\frac{\lambda_I}{\alpha_C}\right)}_{\text{technology asymmetry}} - \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\lambda_K}_{\text{labor specialization}}\right).$$

Proof of Proposition 1.1. In steady state, capital accumulation requires $\mathrm{d}\log I = \mathrm{d}\log K$ and capital valuation requires $\mathrm{d}\log p_K = \mathrm{d}\log r_K + \mathrm{d}\log(1-\tau)$. Substituting the intratemporal solutions gives the long-run supply and demand curves for capital:

$$\mathrm{d}\log K = \frac{\varepsilon_{I,p}}{1-\varepsilon_{I,K}}\,\mathrm{d}\log p_K \qquad \text{(supply)},$$

$$\mathrm{d}\log p_K = \frac{\varepsilon_{r,K}}{1-\varepsilon_{r,p}}\,\mathrm{d}\log K + \frac{1}{1-\varepsilon_{r,p}}\,\mathrm{d}\log(1-\tau) \qquad \text{(demand)}.$$

The supply curve collects the (p_K, K) pairs satisfying the intratemporal block together with $I = \delta K$. The demand curve collects the (p_K, K) pairs satisfying the intratemporal block together with $p_K = (1-\tau)r_K/(r+\delta)$.

Eliminating $\mathrm{d}\log K$ and solving:

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{1-\varepsilon_{I,K}}{(1-\varepsilon_{I,K})(1-\varepsilon_{r,p})-\varepsilon_{I,p}\varepsilon_{r,K}}, \qquad \frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{\varepsilon_{I,p}}{(1-\varepsilon_{I,K})(1-\varepsilon_{r,p})-\varepsilon_{I,p}\varepsilon_{r,K}}.$$

Substituting the explicit formulas for the four intratemporal coefficients from (A.1)–(A.2) yields the closed-form expressions in Proposition 1.1. The key simplification uses

$$\lambda_I(\alpha_C-\alpha_I) + (1-\alpha_C)\lambda_K = \alpha_C\lambda_L, \tag{A.3}$$

which follows from $\lambda_K = \alpha_C\lambda_C + \alpha_I\lambda_I$. Beyond (A.3), the final substitution uses only the share identities $\lambda_C + \lambda_I = 1$ and $\lambda_K + \lambda_L = 1$ and the steady-state relation $\lambda_I/\lambda_K = \delta/(r+\delta)$. □

The four intratemporal coefficients can be expressed in terms of the elasticity of investment to capital at fixed prices:

$$\varepsilon_{I,K} = \frac{(1-\alpha_C)\lambda_K\left(\alpha_I\lambda_L - (\alpha_C-\alpha_I)\lambda_C\,\phi\right)}{\alpha_I(1-\alpha_C)\lambda_L + (\alpha_C-\alpha_I)\lambda_C\lambda_L + \phi\lambda_I\lambda_C(\alpha_C-\alpha_I)^2}. \tag{A.4}$$

The remaining coefficients are

$$\varepsilon_{I,p} = \frac{\varepsilon_{I,K} - \alpha_I}{\alpha_I - \alpha_C}, \qquad \varepsilon_{r,p} = \frac{\lambda_I}{\lambda_K}\,\varepsilon_{I,K}, \qquad \varepsilon_{r,K} = \frac{(\alpha_I - \alpha_C)\lambda_I}{\lambda_K}\,\varepsilon_{I,K} - (1-\alpha_C). \tag{A.5}$$

As in (A.2), the expression for $\varepsilon_{I,p}$ assumes $\alpha_C \neq \alpha_I$ and extends to $\alpha_C = \alpha_I$ as a finite limit. Consumption is pinned down by Hulten's theorem:

$$\mathrm{d}\log C = \varepsilon_{C,p}\,\mathrm{d}\log p_K + \varepsilon_{C,K}\,\mathrm{d}\log K, \qquad \varepsilon_{C,p} = -\frac{\lambda_I}{\lambda_C}\varepsilon_{I,p}, \qquad \varepsilon_{C,K} = \frac{\lambda_K - \lambda_I\varepsilon_{I,K}}{\lambda_C}. \tag{A.6}$$

Proposition 1.1 also gives the exact mapping between price absorption and the dampening of capital accumulation:

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{\lambda_L}\left(\frac{\lambda_K}{\alpha_C} - \lambda_C\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)}\right). \tag{A.7}$$

Incidence. Unit-cost pricing equates weighted changes in output and factor prices:

$$\lambda_I\,\mathrm{d}\log p_K = \lambda_K\,\mathrm{d}\log r_K + \lambda_L\,\mathrm{d}\log w, \tag{A.8}$$

where $\mathrm{d}\log w$ is the income-share-weighted average of consumption- and investment-sector wage changes. In the long run, capital valuation gives $\mathrm{d}\log p_K = \mathrm{d}\log r_K + \mathrm{d}\log(1-\tau)$. Substituting into (A.8) yields

$$\mathrm{d}\log w = \frac{\lambda_K}{\lambda_L}\,\mathrm{d}\log(1-\tau) - \frac{\lambda_K - \lambda_I}{\lambda_L}\,\mathrm{d}\log p_K. \tag{A.9}$$

Thus price absorption tilts incidence toward capital holders. The change in the net income earned by capital holders who maintain their stock of capital constant is

$$\mathrm{d}\log\left((1-\tau)r_K\right) - \frac{\lambda_I}{\lambda_K}\,\mathrm{d}\log p_K = \frac{\lambda_K - \lambda_I}{\lambda_K}\,\mathrm{d}\log p_K. \tag{A.10}$$

Transition dynamics. Using (A.1) and (A.2), the capital valuation and accumulation equations become the 2×2 system

$$Ax_{t+1} = Bx_t + Cz_{t+1}, \tag{A.11}$$

where $x_t = (\mathrm{d}\log p_{K,t}, \mathrm{d}\log K_t)'$ and $z_t = \mathrm{d}\log(1-\tau_t)$. Saddle-path stability requires one eigenvalue of $A^{-1}B$ inside and one outside the unit circle.

Proposition A.1 (Saddle-path stability)**.** *Let* $P(\eta) = \det(B - \eta A)$ *for the matrices in* (A.13)*. Un-*

der the steady-state restrictions, $P(1) < 0$ and the generalized eigenvalues satisfy $\eta_1\eta_2 = 1/\beta > 1$. Therefore, the balanced growth path is saddle-path stable if and only if

$$P(-1) = \det(A + B) > 0. \tag{A.12}$$

This condition is satisfied in every calibration reported in the paper.

Proof. The matrices in (A.11) are

$$\begin{aligned} A &= \begin{pmatrix} \beta(1-\delta) + \beta(r+\delta)\varepsilon_{r,p} - \frac{\varepsilon_{C,p}}{\psi} & \beta(r+\delta)\varepsilon_{r,K} - \frac{\varepsilon_{C,K}}{\psi} \\ 0 & 1 \end{pmatrix}, \\ B &= \begin{pmatrix} 1 - \frac{\varepsilon_{C,p}}{\psi} & -\frac{\varepsilon_{C,K}}{\psi} \\ \delta\varepsilon_{I,p} & 1 - \delta(1-\varepsilon_{I,K}) \end{pmatrix}, \qquad C = \begin{pmatrix} -\beta(r+\delta) \\ 0 \end{pmatrix}. \end{aligned} \tag{A.13}$$

The first row is the capital valuation equation; the Euler equation contributes the $1/\psi$ terms through (A.6). These terms enter A and B symmetrically, so $A - B$ is independent of ψ.

Define

$$\Delta = (1 - \varepsilon_{I,K})(1 - \varepsilon_{r,p}) - \varepsilon_{I,p}\varepsilon_{r,K}.$$

Direct calculation gives

$$P(1) = \det(B - A) = -\beta(r+\delta)\delta\Delta, \qquad \eta_1\eta_2 = \frac{\det(B)}{\det(A)} = \frac{1}{\beta} > 1. \tag{A.14}$$

To sign Δ, let $s = \lambda_I/\lambda_K = \delta/(r+\delta) \in (0,1)$ and let D_0 denote the denominator in (A.4). The steady-state share identities $\lambda_C + \lambda_I = 1$, $\lambda_K + \lambda_L = 1$, $\lambda_K = \alpha_C\lambda_C + \alpha_I\lambda_I$, and $\lambda_I = s\lambda_K$ then imply

$$\Delta = \frac{\alpha_C(1-\alpha_I)(1+\phi)(1-s\alpha_I)\,(1-\alpha_C+s(\alpha_C-\alpha_I))}{D_0\,(1+s(\alpha_C-\alpha_I))^2} > 0.$$

Here $D_0 > 0$: its first two terms combine to

$$\lambda_L \frac{\alpha_C\,((1-\alpha_I) - s\alpha_I(\alpha_C-\alpha_I))}{1+s(\alpha_C-\alpha_I)},$$

and its remaining term is nonnegative. Thus $P(1) < 0$. Since the root product exceeds one, exactly one root lies inside the unit circle if and only if $P(1)P(-1) < 0$. Because $P(1) < 0$, this is equivalent to $P(-1) > 0$. □

When stability holds, capital is predetermined and prices jump to the saddle path. The impact response is

$$\frac{\mathrm{d}\log p_{K,0}}{\mathrm{d}\log p_{K,\infty}} = \frac{1-\eta_s}{\delta(1-\varepsilon_{I,K})}, \tag{A.15}$$

where $\eta_s \in (0,1)$ is the stable eigenvalue. To see this, use the accumulation row

$$\mathrm{d}\log K_{t+1} = (1-\delta(1-\varepsilon_{I,K}))\ \mathrm{d}\log K_t + \delta\varepsilon_{I,p}\ \mathrm{d}\log p_{K,t}. \tag{A.16}$$

Along the saddle path, deviations from the new steady state decay at rate η_s. Matching coefficients on η_s^t and using the long-run supply curve gives (A.15). Asset prices overshoot their long-run level when convergence along the saddle path is faster than capital mean reversion at fixed prices.

Figure A1 summarizes the transition: the demand shift moves the economy from A to B on impact because capital is predetermined, after which capital accumulates along the saddle path toward the new steady state C.

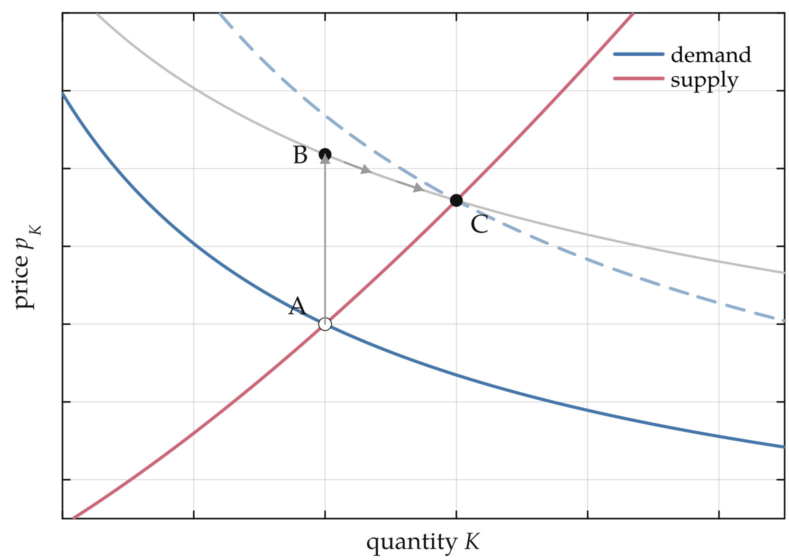


Figure A1: Phase diagram: transition dynamics after a permanent demand shock

A.2 CES production

Start from the stylized model and replace Cobb–Douglas production with CES production in each sector. Let σ_C and σ_I denote the capital–labor elasticities of substitution. The unit-cost equations are unchanged to first order:

$$
\begin{aligned}
0 &= \alpha_C \,\mathrm{d}\log r_K + (1-\alpha_C)\,\mathrm{d}\log w_C, \\
\mathrm{d}\log p_K &= \alpha_I \,\mathrm{d}\log r_K + (1-\alpha_I)\,\mathrm{d}\log w_I.
\end{aligned}
$$

Only factor demands change:

$$
\begin{aligned}
\mathrm{d}\log K_C &= \mathrm{d}\log C + (1-\alpha_C)\sigma_C \,\mathrm{d}\log \frac{w_C}{r_K}, \\
\mathrm{d}\log L_C &= \mathrm{d}\log C + \alpha_C\sigma_C \,\mathrm{d}\log \frac{r_K}{w_C}, \\
\mathrm{d}\log K_I &= \mathrm{d}\log I + (1-\alpha_I)\sigma_I \,\mathrm{d}\log \frac{w_I}{r_K}, \\
\mathrm{d}\log L_I &= \mathrm{d}\log I + \alpha_I\sigma_I \,\mathrm{d}\log \frac{r_K}{w_I}.
\end{aligned}
$$

The labor-supply and intertemporal equations are the same as in the stylized model. Combining CES labor demand with the unchanged Roy labor-supply equations and the unit-cost equations gives the two changed labor-market-clearing equations

$$
\begin{aligned}
\phi \,\mathrm{d}\log \frac{w_C}{w} + \sigma_C \,\mathrm{d}\log w_C &= \mathrm{d}\log C, \\
\phi \,\mathrm{d}\log \frac{w_I}{w} + \sigma_I \,\mathrm{d}\log w_I &= \mathrm{d}\log I + \sigma_I \,\mathrm{d}\log p_K.
\end{aligned}
$$

In the long run, the CES extension has closed-form elasticities:

$$
\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_C-\alpha_I}{1-\alpha_I} + \frac{1-\alpha_C}{\alpha_C(\sigma_C+\phi)(1-\alpha_I)}\left(\sigma_C\alpha_C - \sigma_I\alpha_I\frac{\lambda_K-\lambda_I}{\lambda_C}\right), \tag{A.17}
$$

$$
\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \sigma_I\frac{\alpha_I}{1-\alpha_I} + \frac{\phi(1-\alpha_C)\lambda_C}{\alpha_C(\sigma_C+\phi)\lambda_L(1-\alpha_I)}\left(\sigma_C\alpha_C - \sigma_I\alpha_I\frac{\lambda_K-\lambda_I}{\lambda_C}\right). \tag{A.18}
$$

The Cobb–Douglas case is nested at $\sigma_C = \sigma_I = 1$, in which case (A.17)–(A.18) reduce to Proposition 1.1. Higher substitution in the investment sector raises the quantity response directly. Higher substitution in consumption makes the consumption sector release inputs more elastically, which raises the quantity response; the long-run price response, however, falls with σ_I and rises with σ_C.

A.3 Multiple capital types

Start from the stylized model and allow N capital types and sectors $s \in \{C, I_1, \ldots, I_N\}$. The Cobb–Douglas domestic block replaces the two pricing equations with

$$
\begin{aligned}
0 &= \sum_{m=1}^{N} \alpha_{CK_m} \, \mathrm{d}\log r_{K_m} + \sum_f \alpha_{CL_f} \, \mathrm{d}\log w_{Cf}, \\
\mathrm{d}\log p_{I_n} &= \sum_{m=1}^{N} \alpha_{I_n K_m} \, \mathrm{d}\log r_{K_m} + \sum_f \alpha_{I_n L_f} \, \mathrm{d}\log w_{I_n f}.
\end{aligned}
$$

The corresponding factor-demand equations are the natural multi-factor Cobb–Douglas versions:

$$
\begin{aligned}
\mathrm{d}\log K_{sn} &= \mathrm{d}\log Y_s + \sum_{m \neq n} \alpha_{sK_m} \, \mathrm{d}\log \frac{r_{K_m}}{r_{K_n}} + \sum_f \alpha_{sL_f} \, \mathrm{d}\log \frac{w_{sf}}{r_{K_n}}, \\
\mathrm{d}\log L_{sf} &= \mathrm{d}\log Y_s + \sum_m \alpha_{sK_m} \, \mathrm{d}\log \frac{r_{K_m}}{w_{sf}} + \sum_{g \neq f} \alpha_{sL_g} \, \mathrm{d}\log \frac{w_{sg}}{w_{sf}}.
\end{aligned}
$$

The CES version uses the same equations after multiplying the capital–labor substitution terms by the relevant σ_s, as in Appendix A.2.

Let

$$
\lambda_I = \sum_n \lambda_{I_n}, \qquad \alpha_{CK} = \sum_n \alpha_{CK_n}, \qquad \alpha_{IK} = \sum_n \frac{\lambda_{I_n}}{\lambda_I} \sum_m \alpha_{I_n K_m},
$$

and define aggregate capital-price and capital-quantity responses using capital-income weights,

$$
\mathrm{d}\log p_K = \sum_n \frac{\lambda_{K_n}}{\lambda_K} \, \mathrm{d}\log p_{K_n}, \qquad \mathrm{d}\log K = \sum_n \frac{\lambda_{K_n}}{\lambda_K} \, \mathrm{d}\log K_n.
$$

For a common long-run shock to all capital types, we decompose the exact calibrated response around the aggregate-share version of Proposition 1.1:

$$
\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_{CK} - \alpha_{IK}}{1 - \alpha_{IK}} + \frac{1}{1 + \sum_f \frac{\lambda_{L_f}}{\lambda_L} \phi_f} \frac{1 - \alpha_{CK}}{\alpha_{CK}} \frac{\lambda_K}{\lambda_C} + \mathrm{Cov}_L + \mathrm{Cov}_K. \tag{A.19}
$$

The first two terms are the stylized-model formulas evaluated at the aggregate shares and the labor-income-weighted average of the implied sectoral-equivalent elasticities ϕ_f. The scalar ϕ_f compresses the occupation cross-elasticity matrix $\Phi_f = (\phi_{foo'})$ of Appendix A.4 and the occupation bundles into the sectoral labor-supply elasticity that reproduces the stylized model's wage pass-through. For education group f, let γ_{sfo} be sector s's occupation bundle ($\alpha_{sL_{fo}} =$

$\alpha_{sL_f}\gamma_{sfo}$), let $\ell_{Cfo} = \lambda_C\alpha_{CL_f}\gamma_{Cfo}$ and $\ell_{Ifo} = \sum_n \lambda_{I_n}\alpha_{I_nL_f}\gamma_{I_nfo}$ be steady-state consumption and investment labor income in occupation o, and stack over occupations the consumption bundle $\gamma_{Cf} = (\gamma_{Cfo})_o$, the income-weighted investment bundle γ_{If} with entries $\ell_{Ifo}/\sum_{o'}\ell_{Ifo'}$, and the investment income shares s_{If} with entries $\ell_{Ifo}/(\ell_{Cfo}+\ell_{Ifo})$. Then

$$\phi_f = \frac{1}{\zeta_f} - 1, \qquad \zeta_f = \left(\gamma_{If} - \gamma_{Cf}\right)' \left(\mathrm{Id} + \Phi_f\right)^{-1} s_{If}, \tag{A.20}$$

where Id is the $O \times O$ identity matrix. When each sector employs a single distinct occupation and Φ_f is generated by a scalar Roy elasticity ϕ, then $\zeta_f = 1/(1+\phi)$ and $\phi_f = \phi$. The two remaining terms restore the components suppressed by that benchmark in a fixed order, which makes (A.19) an identity rather than an approximation.

First collapse the investment sectors and capital types to one capital type, while preserving the aggregate consumption and investment occupation factor shares. Impose the internally consistent scalar valuation relation from Proposition 1.1, but solve the full education-by-occupation supply block rather than replacing it by the labor-income-weighted average of ϕ_f. The labor correction is the resulting capital-price response less the first two terms:

$$\begin{aligned}\mathrm{Cov}_L \equiv\ & \left.\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)}\right|_{N=1,\ \text{occupation block}} - \frac{\alpha_{CK}-\alpha_{IK}}{1-\alpha_{IK}} \\ & - \frac{1}{1+\sum_f \frac{\lambda_{L_f}}{\lambda_L}\phi_f}\frac{1-\alpha_{CK}}{\alpha_{CK}}\frac{\lambda_K}{\lambda_C}.\end{aligned} \tag{A.21}$$

Thus this correction includes all occupation-pair elasticities, education supply, and fixed land rather than a scalar approximation by education.

Next restore all capital types and investment sectors. Long-run accumulation and investment q imply $\mathrm{d}\log K_n = \mathrm{d}\log I_n$ and $\mathrm{d}\log p_{K_n} = \mathrm{d}\log p_{I_n}$. Let $\lambda_{W_n} = \sum_s \lambda_{W_{sn}}$ denote the model-implied steady-state installed-capital value share for capital type n. The calibrated valuation equation then gives

$$\mathrm{d}\log r_{K_n} = \left(\frac{\lambda_{I_n}}{\lambda_{K_n}} + \frac{r}{1+r}\frac{\lambda_{W_n}}{\lambda_{K_n}}\right)\mathrm{d}\log p_{I_n} - \mathrm{d}\log(1-\tau). \tag{A.22}$$

Conditional on $(\lambda_{K_n}, \lambda_{I_n}, \lambda_{W_n})$, adjustment costs have no independent long-run effect. Using the steady-state identity $\lambda_{W_n} = (1+r)\lambda_{K_n}/\left(r+(1-\theta_n)\delta_n\right)$ recovers the equivalent coefficient $\lambda_{I_n}/\lambda_{K_n} + r/\left(r+(1-\theta_n)\delta_n\right)$. Thus changing θ_n while recomputing λ_{W_n} changes the steady-state linearization point rather than only the adjustment speed. The coefficient equals one when the scalar model's steady-state user-cost identity holds. The quantitative calibration computes λ_{W_n} from this identity and does not use an empirical market value for capital type n.

For Cobb–Douglas production, capital demand and market clearing therefore imply, for every capital type n,

$$\begin{aligned} \mathrm{d}\log I_n &= \frac{\alpha_{CK_n}\lambda_C}{\lambda_{K_n}}\left(\mathrm{d}\log C - \mathrm{d}\log r_{K_n}\right) \\ &\quad + \sum_m \frac{\alpha_{I_mK_n}\lambda_{I_m}}{\lambda_{K_n}}\left(\mathrm{d}\log I_m + \mathrm{d}\log p_{I_m} - \mathrm{d}\log r_{K_n}\right). \end{aligned} \tag{A.23}$$

Likewise, a worker occupation cell (f, o) satisfies

$$\begin{aligned} \mathrm{d}\log L_{fo} &= \frac{\alpha_{CL_{fo}}\lambda_C}{\lambda_{L_{fo}}}\left(\mathrm{d}\log C - \mathrm{d}\log w_{fo}\right) \\ &\quad + \sum_m \frac{\alpha_{I_mL_{fo}}\lambda_{I_m}}{\lambda_{L_{fo}}}\left(\mathrm{d}\log I_m + \mathrm{d}\log p_{I_m} - \mathrm{d}\log w_{fo}\right). \end{aligned} \tag{A.24}$$

Equations (A.22)–(A.24), the sector unit-cost equations, the long-run occupation and education-supply equations, and fixed sectoral land quantities form a square linear system. They are the exact long-run reduction of the dynamic system; transition speeds and adjustment speeds drop out.

The capital correction is the difference between this full solution and the one-capital occupation solution used in (A.21):

$$\mathrm{Cov}_K \equiv \left.\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)}\right|_{N \text{ capital types, occupation block}} - \left.\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)}\right|_{N=1, \text{ occupation block}}. \tag{A.25}$$

This term restores the capital-type composition, the network of investment sectors, the type-specific valuation coefficients in (A.22), and their interaction with occupation demand. The terms Cov_L and Cov_K are sequential exact corrections rather than standalone covariance formulas. By construction, all four terms in (A.19) add exactly to the calibrated long-run response.

A.4 Dynamic multi-occupation labor supply

This extension starts from the stylized labor block but replaces the scalar sectoral labor-supply curve with an occupation-transition graph. Fix education group f and suppress f on quantities, prices, and occupation bundles. Let occupations be indexed by $o = 1, \ldots, O$, let γ_{so} be sector s's occupation bundle, and let $\phi_{foo'}$ be the long-run elasticity of occupation-o employment with respect to the occupation-o' wage. The elasticities satisfy

$$\sum_{o'=1}^{O} \phi_{foo'} = 0 \quad \text{for each } o, \qquad \sum_{o=1}^{O} \lambda_{L_o} \phi_{foo'} = 0 \quad \text{for each } o'.$$

Within each sector, occupation demands are Cobb–Douglas:

$$\mathrm{d}\log L_{so} = \mathrm{d}\log Y_s + \sum_m \alpha_{sK_m} \mathrm{d}\log \frac{r_{K_m}}{w_{so}} + \sum_{o' \neq o} \alpha_{sL_{o'}} \mathrm{d}\log \frac{w_{so'}}{w_{so}}.$$

In the baseline occupation model, workers are mobile across sectors within an occupation, so $w_{Co} = w_{I_n o} \equiv w_o$ and $p_{L_{Co}} = p_{L_{I_n o}} \equiv p_{L_o}$. The dynamic occupation equation is

$$\mathrm{d}\log L_{o,t+1} = (1-\mu)\,\mathrm{d}\log L_{o,t} + \mu \sum_{o'=1}^{O} \phi_{foo'} \,\mathrm{d}\log p_{L_{o'},t}, \qquad o = 1, \ldots, O, \tag{A.26}$$

where variables are measured relative to their occupation-share-weighted aggregate:

$$\sum_{o=1}^{O} \lambda_{L_o} \mathrm{d}\log L_{o,t} = 0.$$

One occupation equation is redundant under this normalization.

The value of choosing occupation o satisfies

$$\mathrm{d}\log p_{L_o,t} = \frac{r+\mu}{1+r} \mathrm{d}\log w_{o,t+1} - \mathrm{d}\log(1+r_{t+1}) + \frac{1-\mu}{1+r} \mathrm{d}\log p_{L_o,t+1}. \tag{A.27}$$

Financial discounting remains governed by r. The coefficient $1-\mu$ appears because the payoff from the current occupation choice persists only until the worker can reoptimize. In the long run, (A.27) implies $\mathrm{d}\log p_{L_o} = \mathrm{d}\log w_o$, and (A.26) becomes

$$\mathrm{d}\log L_o = \sum_{o'=1}^{O} \phi_{foo'} \,\mathrm{d}\log w_{o'}, \qquad o = 1, \ldots, O. \tag{A.28}$$

Identical occupation bundles imply high sectoral mobility because sectoral reallocation does not require changing occupations. Very different occupation bundles imply lower sectoral mobility because expanding one sector requires moving labor across occupations according to the transition graph. The quantitative model solves (A.26)–(A.27) directly.

A.5 Capital adjustment costs and sector-specific capital

Start from the capital accumulation equation in the stylized model and replace linear installation with the normalized Cobb–Douglas installation technology

$$K_{n,t+1} = (1-\delta_n)K_{n,t} + \delta_n^{\theta_n} K_{n,t}^{\theta_n} I_{n,t}^{1-\theta_n}.$$

The normalization keeps the steady-state investment-capital ratio equal to δ_n. Equating the marginal installed value to the purchase price gives

$$\frac{p_{K_n,t}}{p_{I_n,t}} = \frac{1}{1-\theta_n}\left(\frac{I_{n,t}}{\delta_n K_{n,t}}\right)^{\theta_n}.$$

The linearized accumulation equation is

$$\mathrm{d}\log K_{n,t+1} = (1-(1-\theta_n)\delta_n)\,\mathrm{d}\log K_{n,t} + (1-\theta_n)\delta_n\,\mathrm{d}\log I_{n,t}. \tag{A.29}$$

The investment–q equation is

$$\mathrm{d}\log I_{n,t} - \mathrm{d}\log K_{n,t} = \frac{1}{\theta_n}\left(\mathrm{d}\log p_{K_n,t} - \mathrm{d}\log p_{I_n,t}\right). \tag{A.30}$$

Let

$$\lambda_{K_n} = \frac{r_{K_n}K_n}{Y}, \qquad \lambda_{I_n} = \frac{p_{I_n}I_n}{Y}, \qquad \lambda_{W_n} = (1+r)\frac{p_{K_n}K_n}{Y} = \frac{1+r}{r+(1-\theta_n)\delta_n}\lambda_{K_n}.$$

The capital valuation equation is

$$\begin{aligned}\mathrm{d}\log p_{K_n,t} = {} & \frac{\lambda_{K_n}}{\lambda_{W_n}}\left(\mathrm{d}\log(1-\tau_{n,t+1}) + \mathrm{d}\log r_{K_n,t+1}\right) - \frac{\lambda_{I_n}}{\lambda_{W_n}}\,\mathrm{d}\log p_{I_n,t+1} \\ & - \mathrm{d}\log(1+r_{t+1}) + \frac{1}{1+r}\,\mathrm{d}\log p_{K_n,t+1}.\end{aligned} \tag{A.31}$$

In the long run, (A.30) gives $\mathrm{d}\log p_{K_n} = \mathrm{d}\log p_{I_n}$, so adjustment costs affect transition dynamics but not the long-run relative price of installed capital.

For the sector-specific capital extension, each sector s owns its own type-n capital stock, accumulated with the same convex adjustment-cost technology:

$$\mathrm{d}\log K_{sn,t+1} = (1-(1-\theta_n)\delta_n)\,\mathrm{d}\log K_{sn,t} + (1-\theta_n)\delta_n\,\mathrm{d}\log I_{sn,t}, \tag{A.32}$$

$$\mathrm{d}\log I_{sn,t} - \mathrm{d}\log K_{sn,t} = \frac{1}{\theta_n}\left(\mathrm{d}\log p_{K_{sn},t} - \mathrm{d}\log p_{I_n,t}\right). \tag{A.33}$$

Define

$$\lambda_{K_{sn}} = \frac{r_{K_{sn}} K_{sn}}{Y}, \qquad \lambda_{I_{sn}} = \frac{\lambda_{K_{sn}}}{\lambda_{K_n}} \lambda_{I_n}, \qquad \lambda_{W_{sn}} = \frac{1+r}{r+(1-\theta_n)\delta_n} \lambda_{K_{sn}}.$$

Then (A.31) applies sector by sector using the sector-specific stock, investment, installed-capital price, rental rate, and value shares defined above.

A.6 Imported inputs

Start from the stylized CES model and add imported inputs M_C and M_I with factor shares α_{CM} and α_{IM}. Let

$$\alpha_{CL} = 1 - \alpha_C - \alpha_{CM}, \qquad \alpha_{IL} = 1 - \alpha_I - \alpha_{IM},$$

and normalize the import price by consumption, so $\mathrm{d}\log p_M = 0$. Import demand is

$$\mathrm{d}\log M_C = \mathrm{d}\log C, \tag{A.34}$$

$$\mathrm{d}\log M_I = \mathrm{d}\log I + \sigma_M \,\mathrm{d}\log p_K. \tag{A.35}$$

The unit-cost equations are

$$\begin{aligned} 0 &= \alpha_C \,\mathrm{d}\log r_K + \alpha_{CL} \,\mathrm{d}\log w_C, \\ \mathrm{d}\log p_K &= \alpha_I \,\mathrm{d}\log r_K + \alpha_{IL} \,\mathrm{d}\log w_I. \end{aligned}$$

The domestic factor-demand equations change by the own-price substitution toward imports:

$$\begin{aligned} \mathrm{d}\log K_C &= \mathrm{d}\log C + \sigma_C \alpha_{CL} \,\mathrm{d}\log \frac{w_C}{r_K} - \alpha_{CM}\sigma_M \,\mathrm{d}\log r_K, \\ \mathrm{d}\log L_C &= \mathrm{d}\log C + \sigma_C \alpha_C \,\mathrm{d}\log \frac{r_K}{w_C} - \alpha_{CM}\sigma_M \,\mathrm{d}\log w_C, \\ \mathrm{d}\log K_I &= \mathrm{d}\log I + \sigma_I \alpha_{IL} \,\mathrm{d}\log \frac{w_I}{r_K} - \alpha_{IM}\sigma_M \,\mathrm{d}\log r_K, \\ \mathrm{d}\log L_I &= \mathrm{d}\log I + \sigma_I \alpha_I \,\mathrm{d}\log \frac{r_K}{w_I} - \alpha_{IM}\sigma_M \,\mathrm{d}\log w_I. \end{aligned}$$

Together with labor supply, these equations give the two revised market-clearing conditions:

$$\mathrm{d}\log C = \phi \,\mathrm{d}\log \frac{w_C}{w} + (\sigma_C(1-\alpha_{CM}) + \alpha_{CM}\sigma_M) \,\mathrm{d}\log w_C, \tag{A.36}$$

$$\mathrm{d}\log I + \sigma_I \,\mathrm{d}\log p_K = \phi \,\mathrm{d}\log \frac{w_I}{w} + (\sigma_I(1-\alpha_{IM}) + \alpha_{IM}\sigma_M) \,\mathrm{d}\log w_I. \tag{A.37}$$

For the long-run formulas, write

$$\tilde{\sigma}_C = \sigma_C(1-\alpha_{CM}) + \alpha_{CM}\sigma_M, \qquad \tilde{\sigma}_I = \sigma_I(1-\alpha_{IM}) + \alpha_{IM}\sigma_M.$$

With $\lambda_K = \alpha_C\lambda_C + \alpha_I\lambda_I$ and $\lambda_L = \alpha_{CL}\lambda_C + \alpha_{IL}\lambda_I$, define the numerator and denominator

$$\begin{aligned} N_M &= \lambda_C\alpha_C\left(\alpha_{IL}(\alpha_C+\alpha_{CL})\tilde{\sigma}_C + \phi(\alpha_C\alpha_{IL} - \alpha_I\alpha_{CL})\right) + \alpha_I\alpha_{CL}(\lambda_I\alpha_{IL} - \lambda_C\alpha_C)\tilde{\sigma}_I, \\ D_M &= \lambda_C\alpha_C\left(\alpha_{IL}(\alpha_C+\alpha_{CL})\tilde{\sigma}_C + \alpha_{CL}(1-\alpha_I)\tilde{\sigma}_I\right. \\ &\qquad\qquad \left.+\phi(\alpha_C\alpha_{IL} - \alpha_I\alpha_{CL} + \alpha_{CL})\right) + \alpha_{CL}\alpha_{IL}(\lambda_I\alpha_I\tilde{\sigma}_I - \lambda_K\sigma_I). \end{aligned}$$

The long-run price and quantity responses are

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{N_M}{D_M}, \tag{A.38}$$

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \left(\phi\left(\frac{1-\alpha_I}{\alpha_{IL}} - \frac{\lambda_I-\lambda_K}{\lambda_L}\right) + \tilde{\sigma}_I\frac{1-\alpha_I}{\alpha_{IL}} - \sigma_I\right)\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} + \phi\left(\frac{\alpha_I}{\alpha_{IL}} - \frac{\lambda_K}{\lambda_L}\right) + \tilde{\sigma}_I\frac{\alpha_I}{\alpha_{IL}}. \tag{A.39}$$

Setting $\alpha_{CM} = \alpha_{IM} = 0$ gives the CES formulas in (A.17)–(A.18). Imports damp domestic factor-price pressure by letting each sector substitute toward a fixed-price input.

A.7 Spender-saver discounted Euler

This extension combines the hand-to-mouth spender block and the discounted Euler equation. Hand-to-mouth agents consume an ω share of fixed-factor income, including labor and land. Savers own reproducible capital and consume the residual. Let C^S denote saver consumption. Aggregate consumption satisfies

$$\lambda_C\,\mathrm{d}\log C_t = \omega\lambda_F\,\mathrm{d}\log w_t^F + (\lambda_C - \omega\lambda_F)\,\mathrm{d}\log C_t^S, \tag{A.40}$$

where $\lambda_F\,\mathrm{d}\log w_t^F$ is the income-share-weighted fixed-factor price change. Saver consumption follows the discounted Euler equation

$$\varrho\;\mathrm{d}\log C_{t+1}^S - \mathrm{d}\log C_t^S = \psi\;\mathrm{d}\log(1+r_{t+1}), \qquad 0 < \varrho \le 1. \tag{A.41}$$

Thus the usual Euler row is written for C^S, while aggregate consumption C is pinned down by (A.40). The representative-agent model is nested at $\omega = 0$ and $\varrho = 1$. The pure spender-saver case has $\varrho = 1$, so the long-run real rate is still pinned down by the saver Euler equation and the long-run supply-side elasticities are unchanged. When $\varrho < 1$, bounded paths imply

$$d\log C_t^S = -\psi \sum_{j=0}^{\infty} \varrho^j \, d\log(1 + r_{t+j+1}),$$

and in a permanent steady state,

$$d\log(1 + r)_\infty = -\frac{1 - \varrho}{\psi} \, d\log C_\infty^S.$$

Thus ω changes how income gains feed into aggregate demand along the transition, while $\varrho < 1$ can also change the long-run real-rate response.

B Appendix for Section 2

B.1 Sources and samples

BEA accounts. The production account begins with the BEA annual Make and Use Tables Before Redefinitions at producer value (U.S. Bureau of Economic Analysis, 2024a). We use the historical summary tables for 1963–1996, the current summary tables for 1997–2024, and the 2007, 2012, and 2017 detailed benchmark tables. We harmonize their classifications to the industries in the BEA–BLS Integrated Industry-Level Production Account (U.S. Bureau of Economic Analysis, 2024b). That account supplies industry gross output, value added, intermediate purchases, labor compensation by broad education group, and output-price growth.

The user-cost construction uses the BEA Fixed Asset Accounts (U.S. Bureau of Economic Analysis, 2025). Tables 2.1, 2.2, 2.4, 2.5, 2.7, and 2.8 report current-cost values and quantity indexes for net stocks, depreciation, and investment by asset type. Tables 3.7E, 3.7S, and 3.7I report equipment, structures, and IPP investment by industry. Table 7.5 supplies government investment for the public-sector robustness check.

The return account uses the Federal Reserve Integrated Macroeconomic Accounts (Board of Governors of the Federal Reserve System, 2026) and the BEA personal-consumption deflator. We combine Table S11.1.i.a for nonfinancial corporations, Table S11.2.i.a for nonfinancial noncorporate businesses, and Table S12.i.a for financial corporations. We do not add the household housing sector in Table S1M.i.a. Housing remains in the BEA production and final-use measurement, but not in the IMA construction.

Worker data. Occupation bundles come from the 1960–2000 decennial Censuses and the 2005–2019 and 2021–2024 American Community Surveys in IPUMS USA (Ruggles et al., 2025). We retain people aged 25–54 with positive wage income and at least 40 weeks worked. Person-weighted wage bills, rather than employment counts, determine shares across industries, education groups, and occupations. When exact weeks are unavailable, we use the midpoint of the reported interval. We double fixed decennial topcodes and repeated state-year ACS maxima. The harmonized 1990 industry and occupation codes permit a stable classification across samples. Annual shares are linearly interpolated between survey years, including 2020; outside the survey range they are held at the nearest observation; each year they are renormalized.

Occupation transitions come from the IPUMS CPS ASEC (Flood et al., 2025). We retain people aged 25–54 with positive ASEC weights and valid education, current occupation, and previous-year occupation. ASEC survey years 1977–2025 therefore measure transitions whose origin years run from 1976 through 2024. High school denotes less than a completed four-year college degree, college denotes a completed four-year degree, and post-college denotes education beyond college. The same definitions are used when the KLEMS college group is divided with CPS wage bills.

B.2 Technology accounting

Final uses and organizational investment. Private consumption combines BEA final-use category F010 with residential private fixed investment F02R. Equipment is F02E and excludes changes in private inventories. Structures use nonresidential private fixed investment F02S from 1997 onward. Before 1997 the summary tables report only total structures F02T, so we preserve its commodity mix, rescale the investment component to private nonresidential structures in Fixed Asset Table 2.7, line 36, and assign the residual to consumption. IPP is F02N.

We capitalize selected professional and computer services, following Corrado et al. (2005, 2009, 2022). The detailed Use Tables determine the capitalized professional-services share from architectural and engineering services, management consulting, advertising, and one half of other professional services, and the computer-services share from computer-systems design. Management-of-companies services are fully capitalized. The 2007, 2012, and 2017 benchmark shares are linearly interpolated. Before 2007, the professional-services share is backcast with a three-year moving average of management-occupation payroll and bounded between one half and 1.1 times its 2007 value; the computer-services share is held at its 2007 value. Both shares are held at their 2017 values after 2017. Purchases by the service's own producing industry are excluded. Reclassified purchases are removed from intermediates, recorded as private or government organizational investment, and added by the same amount to value added.

Management of companies was not reported separately before the 1987–1988 treatment of corporate auxiliaries. We therefore set its pre-1988 KLEMS labor share to its 1988–1997 mean. Because the harmonized Census industry classification has no headquarters industry, managers and executives outside government are assigned to management of companies when its occupation bundle is constructed. Brand/Org is therefore management intensive by construction. Figure 2 displays the resulting pre-1988 backcast in a lighter shade.

Industry factor payments. KLEMS supplies industry labor compensation. CPS wage bills divide it into high-school, college, and post-college labor. Census and ACS wage bills then divide each education group among occupations. We use a summary-industry occupation distribution when every education group has positive support, a broad-sector distribution otherwise, and the national distribution only when the broad sector also lacks support.

Land income is 18 percent of the value-added base in agriculture and 24 percent in residential real estate, capped by value added net of labor; it is zero elsewhere. Reproducible-capital income is the residual after intermediate purchases, labor, and land. Thus equipment, structures, IPP, and organizational capital are not separately inferred from industry residuals.

Input–output steps. Write U for the commodity-by-industry Use Table, M for the industry-by-commodity Make Table, and Ω for direct industry factor shares, with reproducible capital still combined. Under the baseline common commodity-technology assumption,

$$D = \left(M' \mathrm{col}\left(M'\right)^{-1}\right)^{-1}, \qquad R = U \,\mathrm{col}\left(M'\right)^{-1} D,$$
$$\Psi = \left(I - R'\right)^{-1} D'\Omega, \qquad \alpha_s' = \beta_s' \Psi.$$

Here D maps commodity production to industries, R gives direct commodity requirements, Ψ gives total factor requirements, and β_s contains sector s's final-use expenditure shares. The alternative common industry-technology assumption uses $D = M \,\mathrm{col}\left(M\right)^{-1}$.

The resulting α_s contains the three education shares, land, and combined reproducible capital. Aggregate factor-income shares weight these sector shares by final-use shares. The time-series construction below divides each sector's combined nonland capital share among equipment, structures, IPP, and organizational capital in proportion to their rental payments. The five annual observations from 2020 through 2024 are averaged before imposing the steady-state identities.

B.3 Labor-reallocation accounting

Occupation bundles. We map the harmonized 1990 occupation codes to the 66 market occupations of Hsieh et al. (2019). Within sector and education group, γ_{sfo} is occupation o's share of the wage bill, so $\alpha_{sL_f}\gamma_{sfo}$ is its factor share. The input–output steps above carry the industry occupation shares into consumption and each investment type. Aggregate investment combines the four investment bundles with their labor payments as weights.

Pairwise flows. For education group f, $P_{f,oo'}$ is the pooled probability that a worker in occupation o last year reports occupation o' this year, and ς_{fo} is the corresponding origin share. With $s_{fo} = \sum_{o'} \varsigma_{fo'} P_{f,o'o}$, the annual occupation-reoptimization rate is

$$\mu_f = \frac{1 - \sum_o \varsigma_{fo} P_{f,oo}}{1 - \sum_o \varsigma_{fo} s_{fo}}.$$

Following Böhm et al. (2025), the relative pattern of own- and cross-elasticities is

$$D_{f,oo'} = \frac{1}{\varsigma_{fo}} \left(\mathbf{1}\left(o = o'\right) s_{fo} - \sum_{\bar{o}} \varsigma_{f\bar{o}} P_{f,\bar{o}o} P_{f,\bar{o}o'} \right).$$

The off-diagonal terms are more negative when two destinations draw workers from the same origins. Each row sums to zero, and each column sums to zero when weighted by origin shares.

The transition data determine relative elasticities but not their common scale. We choose one scale for the occupation classification so that the all-worker average own elasticity is one, consistent with the Roy–Fréchet benchmark in Hsieh et al. (2019, Table II), and apply it unchanged to the three education groups. We then preserve each pairwise-flow conductance while reweighting the matrix from CPS origin shares to the model's occupation-income shares. The resulting $\phi_{foo'}$ satisfies the adding-up restrictions in Appendix A.4. The quantitative model uses the 66-occupation matrices; the broad groups below are displays, not replacement inputs. Table A1 displays the education-specific scaled elasticities in broad occupation groups.

Scale-free specialization. The descriptive specialization index in Section 2 uses the same pairwise transition flows and the occupation wage bills implied by α and γ. For each origin, we multiply its transition shares into every pair of destinations and weight by the origin's population share. We normalize each destination pair by the geometric mean of its two same-destination terms, then compare the consumption and investment bundles under this normalized inner product. For education group f, the index is

$$1 - \frac{g_C' K_f g_I}{\sqrt{\left(g_C' K_f g_C\right)\left(g_I' K_f g_I\right)}}, \qquad K_{f,oo'} = \frac{S_{f,oo'}}{\sqrt{S_{f,oo} S_{f,o'o'}}}, \qquad S_{f,oo'} = \sum_{\bar{o}} \varsigma_{f\bar{o}} P_{f,\bar{o}o} P_{f,\bar{o}o'},$$

where g_C and g_I collect the two bundles' occupation wage-bill shares within group f, $S_{f,oo'}$ is the shared-origin term appearing in $D_{f,oo'}$ above, and K_f is its normalized kernel. The index is zero for identical bundles and for the random-transition benchmark with uniform destination flows, and one when there is no direct or flow-mediated occupational overlap. A common rescaling of the pairwise-flow matrix leaves it unchanged. The construction is applied separately by education group and to each investment type; aggregate investment uses the same labor-payment weights as the production account.

B.4 User costs and returns

Stocks, depreciation, and acquisition value. For equipment, structures, and IPP, annual depreciation is BEA quantity depreciation divided by the lagged quantity stock. We average its smoothed value over 2020–2024 and set organizational capital's rate equal to the IPP rate. Organizational capital follows the same linear perpetual-inventory method as the measured types and is initialized from balanced growth.

Equipment, structures, and IPP remain at their BEA private current-cost net stocks, which are their observed acquisition values. The corresponding quantity indexes separately measure physical stocks, investment, depreciation, and growth. IO/KLEMS supplier-price growth enters the relative investment-price series, deflates the organizational-capital investment series in the perpetual-inventory construction, and values the resulting stock. Make-table market shares allocate commodity purchases to producing industries, and adjacent-year supplier expenditure shares weight KLEMS output-price growth. Investment-price measurement excludes equipment inventories. It uses nonresidential structures from 1997 onward and applies the Fixed Asset Table 2.7 nonresidential control to the historical F02T commodity mix. Each type's aggregate acquisition-value stock is allocated across sectors with that type's nominal investment shares. The nonlinear installation technology is used only in model counterfactuals.

The non-housing IMA claim. The payout flow is after-tax operating surplus less gross investment for nonfinancial corporations, nonfinancial noncorporate businesses, and financial corpo-

Table A1: Occupation labor-supply cross-elasticities $\phi_{foo'}$ (broad occupation groups)

Affected occupation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Panel A: High school																			
1. Exec./admin./managerial	94	-7	-2	-3	0	-2	0	-2	-2	-16	-18	-2	-11	-5	-10	-2	-3	-3	-4
2. Management related	-26	127	-4	-4	0	-2	0	-2	-2	-19	-41	-2	-7	-2	-7	-1	-2	-3	-3
3. STEM	-11	-6	87	-5	0	-1	0	-1	-14	-7	-12	-1	-4	-2	-11	-2	-2	-3	-3
4. Science/arts	-15	-5	-5	148	0	-7	0	-13	-8	-17	-22	-2	-28	-4	-9	-2	-4	-5	-4
5. Doctors/lawyers	-76	-18	-12	-24	915	-157	-1	-14	-78	-91	-182	-16	-69	-16	-43	-21	-34	-29	-35
6. Health services	-4	-1	0	-2	-1	95	0	-2	-18	-10	-13	-1	-26	-2	-2	-2	-3	-4	-2
7. Postsec. teachers	-29	-8	-2	-8	-1	-18	220	-12	-13	-14	-25	-5	-32	-5	-14	-4	-10	-12	-9
8. Teachers/librarians	-10	-3	-1	-15	0	-6	-1	122	-3	-14	-25	-2	-22	-2	-5	-2	-4	-4	-3
9. Health/sci. techs.	-6	-2	-7	-4	-1	-25	0	-2	113	-8	-14	-2	-7	-2	-8	-14	-4	-5	-5
10. Sales	-13	-4	-1	-3	0	-4	0	-2	-2	110	-26	-2	-19	-3	-9	-3	-4	-8	-5
11. Admin. support	-9	-6	-1	-2	0	-4	0	-2	-3	-16	77	-1	-13	-2	-4	-2	-3	-6	-4
12. Protective services	-7	-2	-1	-2	0	-3	0	-1	-2	-7	-10	81	-10	-3	-9	-2	-5	-8	-8
13. Food/cleaning/personal	-8	-1	-1	-4	0	-10	0	-3	-2	-16	-18	-2	112	-9	-10	-3	-8	-11	-6
14. Farm/extraction	-12	-1	-1	-2	0	-3	0	-1	-2	-9	-9	-2	-31	145	-28	-5	-10	-16	-13
15. Mechanics/construction	-6	-1	-1	-1	0	-1	0	-1	-2	-7	-5	-2	-9	-7	76	-4	-7	-12	-12
16. Precision mfg.	-5	-1	-1	-1	0	-3	0	-1	-14	-8	-8	-1	-9	-5	-16	128	-24	-26	-5
17. Mfg. operators	-5	-1	0	-1	0	-3	0	-1	-2	-8	-9	-2	-17	-6	-16	-15	129	-34	-8
18. Fabricators/handlers	-4	-1	-1	-1	0	-3	0	-1	-2	-12	-15	-3	-19	-8	-24	-14	-29	165	-29
19. Vehicle operators	-5	-1	-1	-1	0	-2	0	-1	-3	-9	-12	-3	-11	-7	-26	-3	-7	-30	120
Panel B: College																			
1. Exec./admin./managerial	77	-12	-6	-8	-1	-2	-1	-5	-2	-15	-12	-1	-4	-2	-3	-1	-1	-1	-1
2. Management related	-26	89	-6	-7	-1	-1	-1	-4	-2	-14	-18	-1	-3	-1	-2	-1	-1	-1	-1
3. STEM	-12	-5	53	-6	-1	0	-1	-2	-9	-4	-5	-1	-2	-1	-3	-1	0	-1	-1
4. Science/arts	-21	-8	-7	114	-1	-4	-1	-18	-6	-10	-13	-1	-13	-2	-2	-1	-2	-1	-1
5. Doctors/lawyers	-136	-94	-63	-69	940	-136	-4	-81	-70	-103	-89	-14	-23	-6	-21	-10	-6	-9	-8
6. Health services	-6	-2	-1	-4	-3	43	-1	-4	-3	-4	-6	0	-7	-1	-1	-1	0	-1	-1
7. Postsec. teachers	-25	-10	-9	-18	-1	-14	195	-34	-23	-14	-18	-1	-11	-3	-6	-2	-3	-1	-1
8. Teachers/librarians	-8	-3	-2	-13	-1	-3	-2	71	-3	-9	-13	-1	-8	-2	-1	-1	-1	-1	-1
9. Health/sci. techs.	-11	-5	-18	-10	-2	-4	-3	-6	102	-7	-12	-1	-5	-1	-3	-8	-2	-2	-3
10. Sales	-23	-10	-3	-6	-1	-2	-1	-7	-2	88	-17	-1	-6	-1	-3	-1	-1	-2	-2
11. Admin. support	-21	-15	-4	-9	-1	-4	-1	-13	-5	-19	113	-2	-8	-1	-2	-1	-1	-3	-2
12. Protective services	-10	-5	-3	-4	-1	-1	0	-6	-3	-7	-10	64	-4	-2	-3	-1	-2	-1	-2
13. Food/cleaning/personal	-21	-7	-4	-25	-1	-14	-2	-22	-6	-20	-25	-2	177	-10	-5	-3	-3	-4	-4
14. Farm/extraction	-33	-7	-6	-11	-1	-4	-1	-15	-5	-15	-15	-3	-32	175	-11	-3	-3	-5	-5
15. Mechanics/construction	-17	-7	-9	-6	-1	-1	-1	-5	-4	-13	-10	-2	-7	-4	107	-4	-3	-6	-7
16. Precision mfg.	-16	-5	-7	-8	-1	-5	-1	-10	-32	-12	-13	-1	-10	-3	-10	176	-17	-24	-2
17. Mfg. operators	-17	-6	-6	-15	-1	-2	-2	-8	-10	-12	-18	-4	-14	-4	-9	-25	203	-42	-8
18. Fabricators/handlers	-14	-7	-7	-9	-1	-3	0	-11	-8	-19	-28	-2	-12	-5	-13	-23	-29	236	-45
19. Vehicle operators	-12	-6	-5	-7	-1	-4	0	-8	-11	-19	-20	-4	-11	-5	-17	-2	-5	-43	180
Panel C: Post-college																			
1. Exec./admin./managerial	71	-12	-6	-11	-6	-2	-4	-9	-2	-8	-6	0	-2	-1	-1	-1	0	0	0
2. Management related	-34	94	-8	-9	-6	-1	-3	-6	-2	-9	-11	-1	-2	-1	-1	0	0	0	0
3. STEM	-13	-6	54	-9	-4	0	-4	-2	-8	-2	-3	0	-1	0	-1	0	0	0	0
4. Science/arts	-20	-6	-8	102	-6	-4	-9	-22	-4	-6	-7	-1	-6	-1	-1	0	0	-1	-1
5. Doctors/lawyers	-8	-3	-3	-5	45	-8	-3	-4	-3	-3	-3	0	-1	0	0	0	0	0	0
6. Health services	-5	-1	-1	-6	-17	55	-4	-8	-2	-3	-3	0	-3	0	0	0	0	-1	0
7. Postsec. teachers	-15	-3	-7	-17	-7	-5	92	-18	-4	-4	-6	-1	-2	-1	-1	0	-1	0	0
8. Teachers/librarians	-8	-2	-1	-11	-3	-3	-5	48	-1	-3	-5	0	-3	-1	0	0	0	0	0
9. Health/sci. techs.	-14	-5	-29	-15	-17	-6	-8	-9	137	-5	-8	-1	-3	0	-2	-6	-2	-2	-2
10. Sales	-32	-12	-4	-11	-7	-4	-4	-13	-2	113	-11	-1	-4	-1	-2	-1	-1	-1	-1
11. Admin. support	-26	-18	-7	-18	-10	-5	-8	-23	-5	-13	150	-1	-6	-1	-2	-1	-1	-3	-2
12. Protective services	-12	-6	-4	-8	-6	-2	-6	-8	-3	-4	-5	75	-4	-1	-3	0	-1	-1	-2
13. Food/cleaning/personal	-33	-8	-8	-44	-5	-14	-8	-49	-7	-15	-18	-2	232	-8	-3	-3	-1	-4	-3
14. Farm/extraction	-39	-11	-4	-28	-9	-5	-13	-28	-3	-19	-9	-2	-29	224	-9	-3	-1	-8	-6
15. Mechanics/construction	-19	-5	-13	-14	-5	-3	-5	-11	-7	-13	-10	-2	-5	-4	138	-2	-2	-10	-8
16. Precision mfg.	-23	-4	-7	-7	-13	-5	-2	-13	-38	-11	-13	-1	-9	-3	-5	188	-17	-18	0
17. Mfg. operators	-35	-11	-9	-17	-3	-6	-12	-18	-15	-11	-13	-2	-7	-2	-6	-26	244	-43	-7
18. Fabricators/handlers	-13	-7	-8	-15	-7	-9	-7	-20	-10	-15	-26	-1	-14	-7	-22	-19	-30	268	-37
19. Vehicle operators	-16	-8	-8	-18	-13	-4	-2	-14	-16	-17	-26	-5	-10	-6	-18	0	-5	-39	225

Notes: Entries are own (diagonal) and cross (off-diagonal) labor-supply elasticities multiplied by 100, from pooled education-specific CPS transitions evaluated at CPS origin shares, under the common scale that sets the all-worker average own elasticity to one. Column numbers index the row occupations. The broad matrices are estimated directly at the 19-group classification for display; the quantitative model uses the education-specific 66-occupation matrices, reweighted to the model's occupation-income shares as described above.

rations. The value account combines net claims on nonfinancial corporations, proprietors' equity and loans of noncorporate businesses, and net claims on financial corporations. Household real estate, imputed housing surplus, and residential investment do not enter this return account. Capitalizing organizational services raises surplus and investment equally and therefore leaves payouts unchanged.

The IMA provides one aggregate business claim, not type-level market values. We use that claim only to estimate the common required return and for aggregate validation; we never allocate it across capital types. Type-specific capital income instead uses acquisition-value stocks and replacement-cost user costs. In model counterfactuals, the price of installed capital is an endogenous marginal shadow price implied by the installation first-order condition, not an allocated component of enterprise value.

Returns and user costs. Let V_t and D_t be the real value and payout of the non-housing claim, and let $v_t = \log (V_t / D_t)$ and $d_t = \log D_t$. The state is

$$z_t = \left(v_t,\ \Delta d_t,\ \Delta \log \left(\frac{p_{I_1,t}}{p_{C,t}} \right),\ \ldots,\ \Delta \log \left(\frac{p_{I_N,t}}{p_{C,t}} \right) \right)'.$$

We estimate the first-order system

$$z_t = a + A z_{t-1} + u_t$$

by ordinary least squares, equation by equation, on 1964–2024, so $a + Az_t$ is the one-year-ahead fitted state. Let e_v, e_d, and e_n select v_t, Δd_t, and the relative price growth of type n from z_t. Log-linearizing the return identity about the mean value-payout ratio $\bar{v}$ over 1995–2024 gives $\rho = (1 + e^{-\bar{v}})^{-1}$ and $\kappa = -\log \rho + (1 - \rho)\,\bar{v}$, so the constants are common across types.

Setting the installation curvature to zero makes the price of installed capital equal the purchase price, and the valuation equation of Appendix A.5 reduces to $p_{I_n,t}\,(1 + r_{t+1}) = r_{K_n,t+1} + (1 - \delta_{n,t+1})\, p_{I_n,t+1}$. Dividing by $p_{I_n,t}$, deflating by consumption, and substituting the fitted state gives the user cost

$$\mathrm{uc}_{n,t+1} = \kappa - v_t + \left(\rho e_v' + e_d' \right) (a + Az_t) + \delta_{n,t+1} - (1 - \delta_{n,t+1})\, e_n' \,(a + Az_t)\,.$$

The one-year horizon matches the rental identity, and because type-level traded claims are unavailable the same required return applies to every type. Free disposal bounds a rental price at zero when the fitted capital gain exceeds the required return plus depreciation; the bound does not bind in the baseline sample. Current-dollar capital income multiplies the real user cost by beginning-of-period acquisition value and current consumption-price inflation.

Growth adjustment. The model is stationary while the data grow. The model's required return is therefore the growth-adjusted value $r = \mathbb{E}r - \bar{g} = 0.0378$, where $\mathbb{E}r = 0.0598$ is the headline expected return above and $\bar{g} = 0.0220$ is trend output growth over the reporting window. Depreciation enters the model as $\delta_n + \bar{g}$, so the steady-state investment rate equals its data counterpart $I_n/K_n = \delta_n + \bar{g}$ and the user cost $r + \delta_n$ is unchanged. The investment–q slope below is likewise evaluated at $I_n/K_n = \delta_n + \bar{g}$. Table 4 reports the unadjusted BEA depreciation rates.

Cummins et al. (1996, Table 7) estimate the Summers investment–q specification $I_{it}/K_{i,t-1} = a + bQ_{it} + e_{it}$ by first-difference IV and report $b = 0.650$ (0.077). Define the empirical inverse investment–q slope $\chi \equiv b^{-1} = 1.54$, which we round to 1.5. Appendix A.5 gives p_{K_n}/p_{I_n} under the installation technology, whose local investment–q slope at $I_n/K_n = \delta_n$ is $\delta_n(1-\theta_n)/\theta_n$. Matching that slope to $1/\chi$ gives $\theta_n = \delta_n\chi/(1+\delta_n\chi)$. The empirical slope is used only for this mapping and does not enter as a separate model parameter; the adjustment technology is not used to construct historical stocks or market values. Table A2 reports the VAR moments and coefficients.

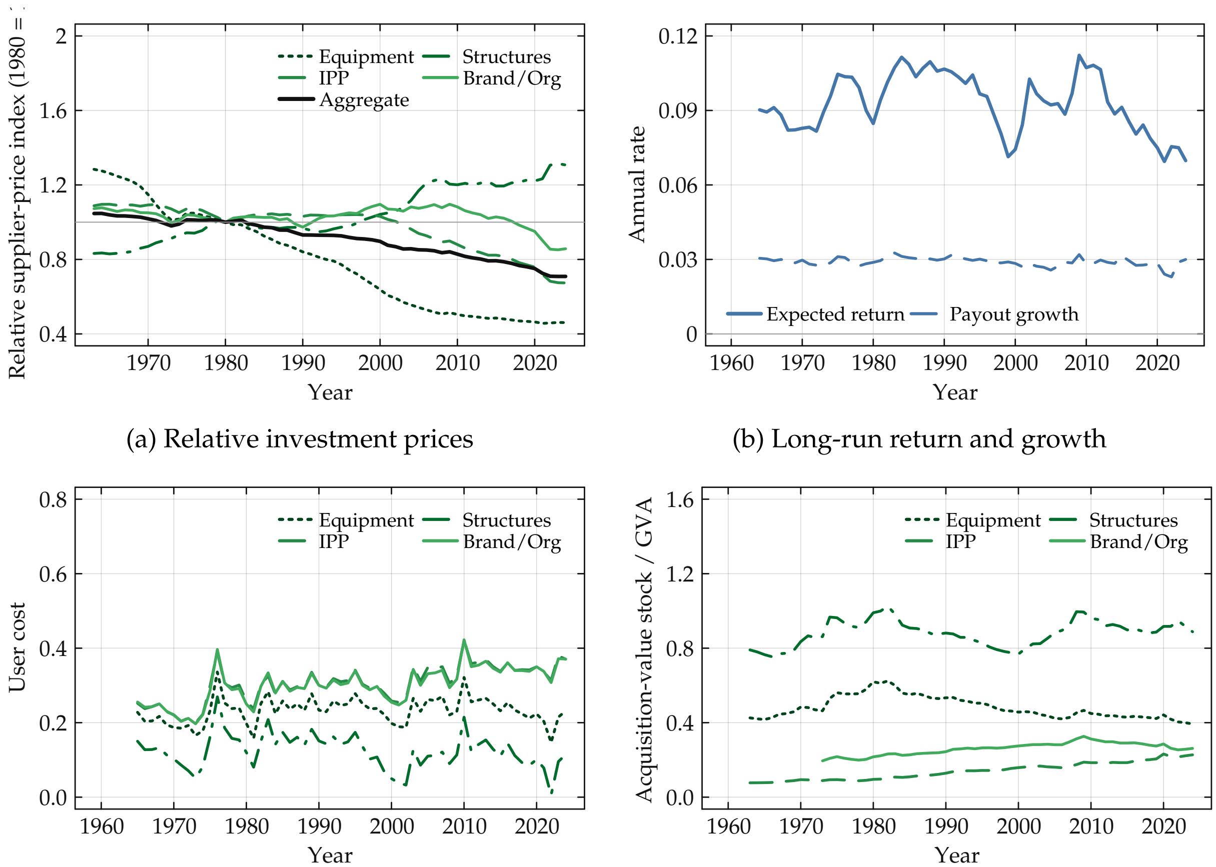
Table A2: Summary statistics and return-price VAR

Variable	N	Mean	SD	Min.	Max.	AR(1)
v_t	61	2.72	0.19	2.44	3.13	0.86
Δd_t	61	2.93	6.52	-12.03	20.40	0.13
$\Delta \log\left(p_{I_{\text{Equipment}},t}/p_{C,t}\right)$	61	-1.68	1.52	-5.04	3.10	0.63
$\Delta \log\left(p_{I_{\text{Structures}},t}/p_{C,t}\right)$	61	0.74	1.53	-3.73	5.76	0.40
$\Delta \log\left(p_{I_{\text{IPP}},t}/p_{C,t}\right)$	61	-0.79	1.36	-5.30	1.94	0.55
$\Delta \log\left(p_{I_{\text{Brand/Org}},t}/p_{C,t}\right)$	61	-0.37	1.64	-5.32	2.84	0.33

$$a = \begin{pmatrix} 0.33 \\ -0.05 \\ 0.02 \\ -0.03 \\ 0.04 \\ 0.07 \end{pmatrix}, \qquad A = \begin{pmatrix} 0.89 & -0.04 & 1.71 & 1.82 & 0.99 & -0.24 \\ 0.03 & 0.08 & -0.14 & -0.94 & -0.06 & 0.51 \\ -0.01 & -0.03 & 0.61 & -0.13 & -0.47 & -0.07 \\ 0.01 & 0.07 & -0.28 & 0.18 & -0.71 & 0.32 \\ -0.01 & -0.07 & 0.03 & 0.11 & 0.68 & -0.03 \\ -0.03 & -0.08 & -0.12 & 0.16 & 0.16 & 0.26 \end{pmatrix}, \qquad \rho = 0.94, \qquad \kappa = 0.22.$$

Notes: The VAR sample is 1964–2024. Growth-rate moments are 100 times annual log changes; AR(1) is the sample first-order autocorrelation. The VAR is estimated in log units; rows and columns of A follow the table order. The Campbell–Shiller constants use the mean value-payout ratio over 1995–2024. See Appendix B.4.

Figure A2 plots the time-series inputs. The stable fitted transition does not outperform the historical mean in expanding-window forecasts, so we use the VAR only as a joint conditional projection. Tables A3 and A4 report annual production shares and their 2020–2024 model averages. Table A5 summarizes all-worker transitions in broad categories; main-text Table 2 shows the occupation bundles, while the calibration uses education-specific flows among 66 occupations.



(a) Relative investment prices

(b) Long-run return and growth

(c) User costs by capital type

(d) Capital stocks by type

Figure A2: User costs and returns

Notes: In Panel (a), supplier-price indexes are relative to consumption, their aggregate is expenditure weighted, and all series are normalized to one in 1980. In Panel (c), user costs are per dollar of beginning-of-period acquisition value. In Panel (d), stocks are relative to business value added; measured types use BEA current-cost net stocks and organizational capital is supplier-price valued.

Table A3: Annual production shares

		Sector share	Capital shares				Labor shares			Land
Year	Sector	$\lambda_{s,t}^{\text{data}}$	Equipment	Structures	IPP	Brand/Org	HS	College	Post-college	Land
2020	C	0.77	0.14	0.17	0.07	0.13	0.23	0.13	0.11	0.03
	$I_{\text{Equipment}}$	0.07	0.15	0.05	0.16	0.15	0.24	0.15	0.10	0.00
	$I_{\text{Structures}}$	0.03	0.18	0.08	0.04	0.13	0.39	0.12	0.06	0.00
	I_{IPP}	0.05	0.08	0.03	0.17	0.10	0.16	0.26	0.20	0.00
	$I_{\text{Brand/Org}}$	0.08	0.08	0.04	0.10	0.10	0.18	0.27	0.24	0.00
Aggregate factor-income shares, λ_j			0.13	0.14	0.08	0.13	0.22	0.15	0.12	0.03
2021	C	0.77	0.14	0.16	0.07	0.13	0.22	0.13	0.11	0.03
	$I_{\text{Equipment}}$	0.07	0.15	0.05	0.16	0.15	0.24	0.15	0.10	0.00
	$I_{\text{Structures}}$	0.03	0.18	0.08	0.05	0.13	0.38	0.12	0.06	0.00
	I_{IPP}	0.05	0.08	0.03	0.17	0.10	0.16	0.26	0.20	0.00
	$I_{\text{Brand/Org}}$	0.08	0.08	0.03	0.11	0.09	0.17	0.28	0.24	0.00
Aggregate factor-income shares, λ_j			0.13	0.13	0.09	0.13	0.22	0.15	0.12	0.03
2022	C	0.77	0.14	0.15	0.08	0.14	0.22	0.13	0.11	0.03
	$I_{\text{Equipment}}$	0.07	0.14	0.05	0.16	0.16	0.24	0.15	0.10	0.00
	$I_{\text{Structures}}$	0.03	0.18	0.07	0.05	0.14	0.38	0.12	0.06	0.00
	I_{IPP}	0.05	0.08	0.03	0.18	0.10	0.16	0.26	0.20	0.00
	$I_{\text{Brand/Org}}$	0.08	0.07	0.03	0.11	0.09	0.17	0.28	0.24	0.00
Aggregate factor-income shares, λ_j			0.13	0.12	0.09	0.13	0.22	0.15	0.12	0.03
2023	C	0.77	0.14	0.14	0.08	0.14	0.22	0.13	0.11	0.03
	$I_{\text{Equipment}}$	0.07	0.14	0.05	0.17	0.15	0.23	0.16	0.10	0.00
	$I_{\text{Structures}}$	0.03	0.17	0.07	0.05	0.15	0.38	0.12	0.06	0.00
	I_{IPP}	0.05	0.08	0.03	0.18	0.09	0.15	0.26	0.20	0.00
	$I_{\text{Brand/Org}}$	0.08	0.07	0.03	0.11	0.09	0.17	0.28	0.24	0.00
Aggregate factor-income shares, λ_j			0.13	0.12	0.10	0.14	0.22	0.15	0.12	0.03
2024	C	0.77	0.14	0.14	0.09	0.14	0.21	0.13	0.11	0.03
	$I_{\text{Equipment}}$	0.07	0.14	0.04	0.17	0.15	0.23	0.16	0.10	0.00
	$I_{\text{Structures}}$	0.03	0.17	0.07	0.05	0.15	0.37	0.12	0.06	0.00
	I_{IPP}	0.05	0.08	0.03	0.18	0.09	0.15	0.27	0.21	0.00
	$I_{\text{Brand/Org}}$	0.08	0.07	0.03	0.11	0.09	0.16	0.29	0.24	0.00
Aggregate factor-income shares, λ_j			0.13	0.12	0.10	0.14	0.21	0.15	0.13	0.03

Notes: $\lambda_{s,t}^{\text{data}}$ are observed final-use shares. Aggregate factor-income shares weight sector factor shares by $\lambda_{s,t}^{\text{data}}$. See Appendix B.2.

Table A4: Production shares used in the model

		Sector shares		Capital shares				Labor shares			Land
Years	Sector	$\bar{\lambda}_s^{\text{data}}$	λ_s	Equipment	Structures	IPP	Brand/Org	HS	College	Post-college	Land
2020–2024	C	0.77	0.64	0.14	0.15	0.08	0.14	0.22	0.13	0.11	0.03
	$I_{\text{Equipment}}$	0.07	0.10	0.14	0.05	0.16	0.15	0.24	0.15	0.10	0.00
	$I_{\text{Structures}}$	0.03	0.07	0.18	0.07	0.05	0.14	0.38	0.12	0.06	0.00
	I_{IPP}	0.05	0.08	0.08	0.03	0.18	0.10	0.16	0.26	0.20	0.00
	$I_{\text{Brand/Org}}$	0.08	0.11	0.07	0.03	0.11	0.09	0.17	0.28	0.24	0.00
Aggregate factor-income shares, λ_j				0.13	0.11	0.10	0.13	0.22	0.16	0.13	0.02

Notes: $\bar{\lambda}_s^{\text{data}}$ are raw 2020–2024 average final-use shares; λ_s impose the steady-state identities. Aggregate factor-income shares weight sector factor shares by λ_s. See Appendix B.2.

Table A5: Broad occupation transition summary

Origin occupation	Origin share	Stay probability	Most common different destination	Probability
Exec./admin./managerial	0.11	0.95	Sales	0.01
Management related	0.04	0.94	Exec./admin./managerial	0.02
STEM	0.04	0.97	Exec./admin./managerial	0.01
Science/arts	0.04	0.93	Exec./admin./managerial	0.01
Doctors/lawyers	0.02	0.98	Health services	0.01
Health services	0.05	0.96	Food/cleaning/personal	0.01
Postsec. teachers	0.01	0.93	Science/arts	0.01
Teachers/librarians	0.05	0.96	Science/arts	0.01
Health/sci. techs.	0.03	0.93	Health services	0.02
Sales	0.10	0.94	Admin. support	0.01
Admin. support	0.14	0.95	Sales	0.01
Protective services	0.02	0.96	Admin. support	0.01
Food/cleaning/personal	0.09	0.93	Admin. support	0.01
Farm/extraction	0.02	0.91	Food/cleaning/personal	0.02
Mechanics/construction	0.09	0.96	Vehicle operators	0.01
Precision mfg.	0.03	0.92	Fabricators/handlers	0.01
Mfg. operators	0.04	0.93	Fabricators/handlers	0.02
Fabricators/handlers	0.05	0.90	Mfg. operators	0.02
Vehicle operators	0.04	0.93	Fabricators/handlers	0.02

Notes: Entries use pooled CPS ASEC person weights. The broad categories are a display; the model uses education-specific transitions among 66 occupations (Appendix B.3).

B.5 Robustness

B.5.1 Technology

Table A6 omits the capitalization of organizational services, expands capitalization to selected financial services, includes government, excludes housing from consumption, and replaces the common commodity-technology assumption with the common industry-technology assumption. The financial-services row also capitalizes part of commodities 521CI and 523. The government row combines BEA government consumption with equipment, structures, IPP, and organizational investment. The housing row removes residential investment and the housing commodity (imputed and market rents) from the consumption bundle; the summary-level tables do not separate owner- from tenant-occupied housing. Every row recomputes the input–output account.

Table A6: Labor share by sector, 2020–2024

	C	I	$I - C$
Panel A: Total labor share			
Baseline	0.45	0.60	+0.15
Without capitalizing organizational services	0.50	0.60	+0.10
Capitalizing financial services	0.43	0.57	+0.14
Including government sector	0.48	0.60	+0.12
Excluding housing from consumption	0.51	0.60	+0.08
Using common industry-technology assumption	0.46	0.58	+0.12
Panel B: College-or-above labor share			
Baseline	0.25	0.41	+0.16
Without capitalizing organizational services	0.29	0.38	+0.09
Capitalizing financial services	0.24	0.39	+0.15
Including government sector	0.28	0.40	+0.12
Excluding housing from consumption	0.29	0.41	+0.11
Using common industry-technology assumption	0.26	0.38	+0.12

Table A7 uses the 402-industry detailed benchmarks in 2007, 2012, and 2017. Table A8 repeats the technology alternatives for the change from the beginning to the end of the sample.

Across these alternatives, the 2020–2024 investment labor share exceeds the consumption labor share by 8–15 percentage points, and the college-or-above gap is 9–16 percentage points. Omitting organizational capitalization narrows both gaps and their increase over time; excluding housing from consumption narrows the level gap the most, to 8 percentage points. The remaining technology choices have smaller effects.

The baseline assigns imports domestic factor shares. The domestic-account alternative allocates each commodity's imports across uses in proportion to total use, subtracts them from the Use Table, and records imports as a separate input. Table A9 reports the result.

Table A7: Summary and detailed input–output accounts

	C	I	Difference
Panel A: Total labor share			
Baseline	0.47	0.59	+0.12
Using detailed input-output tables	0.47	0.59	+0.12
Panel B: College-or-above labor share			
Baseline	0.23	0.35	+0.12
Using detailed input-output tables	0.23	0.34	+0.11

Table A8: Change in labor shares, 1963–1968 to 2020–2024

	Consumption	Investment	Difference
Panel A: Total labor share			
Baseline	−0.08	−0.02	+0.06
Without capitalizing organizational services	−0.05	−0.02	+0.03
Capitalizing financial services	−0.09	−0.02	+0.06
Including government sector	−0.07	−0.03	+0.04
Excluding housing from consumption	−0.07	−0.02	+0.05
Using common industry-technology assumption	−0.08	−0.04	+0.04
Panel B: College-or-above labor share			
Baseline	+0.17	+0.31	+0.14
Without capitalizing organizational services	+0.20	+0.29	+0.08
Capitalizing financial services	+0.16	+0.29	+0.14
Including government sector	+0.16	+0.30	+0.13
Excluding housing from consumption	+0.20	+0.31	+0.11
Using common industry-technology assumption	+0.17	+0.28	+0.11

Table A9: Domestic factor shares by sector, 2020–2024

Sector	Factor shares								
	Labor			Land	Capital				Import
	HS	College	Post-college		Equipment	Structures	IPP	Brand/Org	
$C+I$	0.20	0.13	0.11	0.03	0.12	0.11	0.08	0.12	0.10
C	0.20	0.12	0.10	0.03	0.12	0.14	0.07	0.12	0.09
I	0.18	0.19	0.15	0.00	0.09	0.03	0.11	0.10	0.15
$I_{\text{Equipment}}$	0.15	0.10	0.06	0.00	0.09	0.03	0.11	0.10	0.35
$I_{\text{Structures}}$	0.36	0.11	0.06	0.00	0.16	0.07	0.04	0.12	0.09
I_{IPP}	0.14	0.25	0.19	0.00	0.07	0.03	0.16	0.09	0.07
$I_{\text{Brand/Org}}$	0.16	0.27	0.23	0.00	0.07	0.03	0.10	0.09	0.06

B.5.2 Labor specialization

Table A10 reports the aggregate-investment specialization series in Figure 2b under the baseline and six alternative measurement choices. The first two columns average the beginning and end of the sample; the last reports the change between them.

Table A10: Occupation specialization under alternative measurement choices

Specification	Specialization index		Change
	1963–1968	2020–2024	Late minus early
Baseline	29.1	28.9	−0.1
19 broad occupations	16.7	23.0	+6.2
Pooled all-worker transitions	29.2	29.1	−0.1
Unweighted transitions	29.1	29.0	−0.2
Equal education weights	43.2	31.3	−11.9
Uniform origin weights	28.4	28.3	0.0
Managers in reporting industries	11.7	10.6	−1.1

Notes: Entries report aggregate-investment specialization relative to consumption on the same zero-to-100 scale as Figure 2b; higher values indicate less occupational overlap. The baseline uses 66 occupations, education-specific person-weighted transitions, wage-bill education weights, and estimated origin weights. Each alternative changes one choice. Broad occupations use 19 categories; pooled transitions use all-worker flows for each education group; unweighted transitions count linked workers equally; equal education weights average the three groups; uniform origin weights give every origin equal weight; and managers in reporting industries skips the baseline assignment of private-sector managers to management of companies, keeping them in the industries where the microdata report them.

Transition weighting, education pooling, and origin weighting barely change the baseline. Broad occupations lower the level and imply a larger rise. Equal education weights instead produce a decline, so the time profile partly reflects changing education wage-bill weights.

C Appendix for Sections 3–4

C.1 Validation details

This section reports the published coefficients underlying Table 6 and defines the corresponding empirical and model shares.

Table A11: Data moments cited in Table 6

Paper	Moment	Source	Value	SE
Goolsbee (1998b)	supply elasticity	Table VII, col. 1	1.141	0.505
Goolsbee (1998a)	log annual income	Table 1, row 2	0.232	0.030
	log hourly wage	Table 1, row 3	0.219	0.032
	log hours last week	Table 1, row 4	0.009	0.013
House and Shapiro (2008)	real-investment elasticity to total cost	Table 4, Panel A, row 1 (WLS baseline)	7.68	1.85
House, Mocanu and Shapiro (2017)	real equipment purchases	Table 3, col. 1	1.94	0.44
	equipment production price	Table 4, Panel A, col. 1	−0.12	0.10
	production-worker hours	Table 4, Panel A, col. 1	0.77	0.45
	production-worker wage bill	Table 4, Panel A, col. 1	1.28	0.91
	real structures investment (= production)	Table 4, Panel B, col. 1	0.87	0.32
	structures investment price	Table 4, Panel B, col. 1	0.54	0.12

Standard errors. Each share is a smooth function of published coefficients, and Table 6 reports its delta-method standard error. Two shares invert a single elasticity, so their standard errors are exact. The remaining four combine two coefficients from the same regression, and no study reports the covariance between them; for those we report the largest standard error consistent with any correlation, which is $|\partial s/\partial a|\,\sigma_a + |\partial s/\partial b|\,\sigma_b$ for a share s of coefficients a and b.

Study-specific constructions. The model comparisons hold the 2020–2024 baseline calibration fixed and reproduce each study's sector and reported outcome pair. Except where a study's own policy sequence is the object of interest, each comparison announces a sustained subsidy one year before it takes effect and averages over the first year in which it is in force.

Goolsbee (1998b). They use annual data for 81 equipment-producing industries over 1959–1988. Column 1 of their Table VII reports an equipment-output response per unit output-price response of 1.141, so output prices account for 0.47 of the nominal response. We use that column rather than their later columns because those add year dummies, which identify the elasticity from variation between assets within years and so remove the economy-wide component that the model's response contains, and because column 1 does not condition on the wage of the capital-producing industry, which is the margin the model's mechanism operates through.

Because the investment tax credit is legislated, the model announces a sustained equipment-demand subsidy one year before it takes effect and averages over the first year in which it is in force.

Goolsbee (1998a). They use 17,700 observations from the 1968–1994 CPS and restrict the sample to full-time white male scientists and engineers, ages 21–65, with at least a college degree. The reported hourly-wage response of 0.219 and weekly-hours response of 0.009 imply that wages account for 0.96 of their combined response. The paper's "about 95 percent" statement instead compares the hourly-wage response with the separately estimated annual-income response of 0.232. Its direct IV hours-supply elasticity of 0.1–0.2 appears only in the text, without a coefficient or standard error, and is therefore not tabulated. The model announces a sustained IPP-demand subsidy one year before it takes effect, averages over the first year in which it is in force, and combines the economy-wide wage and labor responses of Engineers and Natural Science occupations for college and post-college workers using baseline labor-income weights. The empirical quantity is weekly hours among incumbent full-time workers, whereas the model has no intensive-hours margin and its quantity is the stock of workers in those occupations.

House and Shapiro (2008). They estimate responses to temporary bonus depreciation using 36 types of equipment, quasi-structures, and structures. Their preferred real-investment elasticity of 7.68 implies that total investment cost accounts for 0.12 of the nominal response; total cost includes both external purchase costs and internal adjustment costs. The model applies the anticipated statutory 30 and 50 percent bonus-depreciation sequence to equipment and projects its investment and cost responses on that sequence.

House, Mocanu and Shapiro (2017). They estimate responses across 28 equipment types and 17 nonresidential structures types. In their preferred specification, equipment purchases rise by 1.94 while the producer price falls by 0.12, giving the reported share of -0.07. Production-worker hours rise by 0.77 and the wage bill by 1.28, so wages account for 0.40 of the payroll response. Structures purchases and prices rise by 0.87 and 0.54, giving a price share of 0.38. The model announces a sustained subsidy to the corresponding investment sector one year before it takes effect and averages over the first year in which it is in force; its payroll measure includes all non-land workers rather than production workers alone.

C.2 Robustness of welfare incidence

Table A12 reports how the welfare-incidence shares at $\tau = 0.20$ vary under the alternative modified Euler equations, supply-side parameters, and data constructions used in Table 5. The first row reports the calibrated baseline in levels. The remaining rows report signed deviations from that baseline in percentage points.

Table A12: Robustness of welfare incidence

	Workers				Capitalists			
	Total	HS	Col.	Post-col.	Total	Tang.	Intan.	Land
Calibrated model	1.01	0.38	0.35	0.27	0.26	0.15	0.06	0.05
Demand-side sensitivities								
intertemporal elasticity $\psi = 0.5$	−6	−3	−2	−1	+5	+3	+2	0
intertemporal elasticity $\psi = 2$	+4	+2	+1	+1	−3	−2	−1	0
spender-saver $\omega = 0.25$	0	0	0	0	0	0	0	0
spender-saver $\omega = 0.50$	−1	0	0	0	0	0	0	0
discounted Euler $\varrho = 0.98$	+1	0	0	0	−1	0	0	0
discounted Euler $\varrho = 0.95$	+3	+1	+1	+1	−2	−1	−1	0
Supply-side extensions								
imported inputs $\sigma_M = 3$	−3	0	−3	−1	+15	+7	+4	+4
imported inputs $\sigma_M = 5$	0	+1	−2	0	+20	+10	+5	+4
capital–labor elasticity $\sigma_{KL} = 0.5$	−8	−1	−4	−3	−5	−4	−2	+1
capital–labor elasticity $\sigma_{KL} = 1.5$	+9	+1	+5	+3	+4	+4	+1	−1
occupation transition speeds $2 \times \mu_f$	0	0	0	0	0	0	0	0
occupation transition speeds $10 \times \mu_f$	0	0	0	0	0	0	0	0
occupation elasticities $2 \times \phi_{foo'}$	0	0	0	0	0	0	0	0
occupation elasticities $0.5 \times \phi_{foo'}$	0	0	0	0	0	0	0	0
Alternative data construction								
no capitalization of org. services	−4	+2	−3	−4	+4	+5	−1	+1
capitalization of financial services	0	+1	0	−1	+2	+1	+1	0
consolidate government sector	+1	0	0	0	0	0	0	−1
common industry-tech. assumption	0	+1	−1	0	+2	+1	+1	0

Notes: Except for the baseline row, entries are percentage-point deviations from the baseline.

C.3 Welfare incidence under optimal non-uniform taxation

Table A13 reports the locally optimal schedules and the associated reductions in the Harberger triangle. For every interior reproducible-capital rate, the local first-order condition is

$$\frac{\frac{1}{2}\sum_{m=1}^{N}\tau_m\sum_{t\geq 0}R^{-t}\left(\lambda_{K_n}\frac{\partial\log K_{n,t+1}}{\partial\log(1-\tau_m)}+\lambda_{K_m}\frac{\partial\log K_{m,t+1}}{\partial\log(1-\tau_n)}\right)}{\sum_{m=1}^{N}\tau_m\lambda_{K_m}\sum_{t\geq 0}R^{-t}\,\mathrm{d}\log K_{m,t+1}}=\frac{\lambda_{K_n}\left(\frac{1+r}{r}+\sum_{t\geq 0}R^{-t}\left(\mathrm{d}\log r_{K_n,t+1}+\mathrm{d}\log K_{n,t+1}\right)\right)}{\sum_{m=1}^{N}\tau_m\lambda_{K_m}\left(\frac{1+r}{r}+\sum_{t\geq 0}R^{-t}\left(\mathrm{d}\log r_{K_m,t+1}+\mathrm{d}\log K_{m,t+1}\right)\right)}.$$

The two sides are type n's shares of the marginal efficiency cost and fiscal effect, respectively.

Table A13: Locally optimal non-uniform capital and land-rent tax rates

	Benchmark uniform rate $\bar{\tau}$					
	0	0.10	0.20	0.30	0.40	0.50
Panel A: Tax rates						
Land rent	0.00	1.00	1.00	1.00	1.00	1.00
Brand/Org	0.00	0.08	0.20	0.32	0.44	0.55
Structures	0.00	0.08	0.19	0.30	0.42	0.53
IPP	0.00	0.07	0.17	0.26	0.36	0.46
Equipment	0.00	0.06	0.15	0.23	0.32	0.40
Panel B: Harberger triangle						
Harberger triangle, uniform	0.00	0.25	0.98	2.21	3.93	6.14
Harberger triangle, optimal	0.00	0.12	0.74	1.87	3.52	5.68
Reduction (%)	–	49.36	24.63	15.29	10.42	7.43

Notes: Schedules satisfy the local conditions in Section 4 and match the uniform benchmark's government-welfare effect. Land is taxed first, up to its limiting rate of 1; triangle reduction is undefined at $\bar{\tau} = 0$.

Table A14 evaluates the same common permanent unit-log tax cut used in Table 7, now around locally optimal schedules constructed as in Table A13 for benchmark uniform rates $\bar{\tau} \in \{0, 0.20, 0.35\}$. Each schedule includes land rents and matches the government's welfare effect under the corresponding uniform rate in absolute units, so each pair of rows uses the same denominator. Worker incidence is unchanged because the reform and equilibrium response are common across types. Capitalist incidence changes with the type-specific pre-existing rates; at the limiting 100% land rate, landowners receive no first-order after-tax gain.

Table A14: Welfare incidence under optimal non-uniform capital and land-rent taxes

	Workers				Capitalists				Government	Aggregate
	Total	HS	Col.	Post-col.	Total	Tang.	Intan.	Land	Total	Total
Panel A: $\tau = 0$										
Uniform rates	0.71	0.27	0.25	0.19	0.29	0.15	0.10	0.05	-1.00	0.00
Optimal non-uniform rates	0.71	0.27	0.25	0.19	0.29	0.15	0.10	0.05	-1.00	0.00
Panel B: $\tau = 0.20$										
Uniform rates	1.01	0.38	0.35	0.27	0.26	0.15	0.06	0.05	-1.00	0.26
Optimal non-uniform rates	1.01	0.38	0.35	0.27	0.22	0.16	0.06	0.00	-1.00	0.23
Panel C: $\tau = 0.35$										
Uniform rates	1.47	0.56	0.52	0.40	0.20	0.15	-0.01	0.06	-1.00	0.67
Optimal non-uniform rates	1.47	0.56	0.52	0.40	0.15	0.16	-0.01	0.00	-1.00	0.63

Notes: Entries are welfare changes divided by the matched net government revenue loss. Aggregate sums workers, capitalists, and government. Panels B and C impose the limiting land-rent rate of 1.